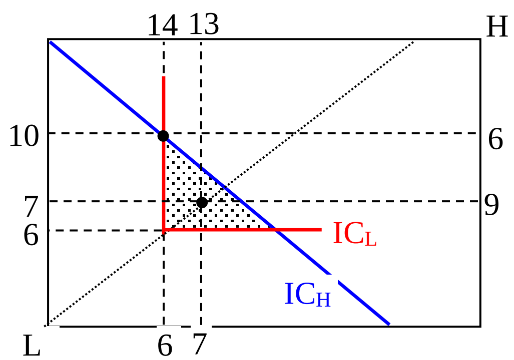
General Equilibrium: Distributive efficiency

1. Suppose Yoosoon and Bob are on an island where there are only coconuts andbananas. The total number of coconuts is 100, total number of bananas is 50. At some allocation it is known that Yoosoon's marginal rate of substitution of bananas for coconuts is $\frac{1}{2}$, while Bob's marginal rate of substitution of bananas for coconuts is 1. Is this allocation Pareto efficient? If not, explain how you can increase efficiency with further trade between Yoosoon and Bob

The marginal rate of substitution tells you how many units of one good a person is willing to give up in order to get one additional unit of the other good and be as well off as before. In our case, Yoosoon is willing to give up $\frac{1}{2}$banana to get one more coconut, while Bob is willing to give up one banana to get one more coconut. This means Bob could give Yoosoon one banana, which is worth two coconuts to her. If she gives Bob 3/2 coconuts in return for one banana, Bob is better off since he only needs one coconut in return for one banana to make him as well off as before. Yoosoon is also better off since she would be willing to give up two coconuts for an additional banana.

2. Lucie is currently consuming 6 glasses of milk and 10 cookies, while Hope is consuming 14 glasses of milk and 6 cookies. Hope views milk and cookies as perfect substitutes (her MRS is 1), while Lucie's preferences over milk and cookies are such that she only likes to eat a cookie if she can also drink a glass of milk (and she only likes milk if she can eat one cookie per glass of milk drunk).

a) Is the current allocation of milk and cookies Pareto efficient? Explain with use of a diagram.



Note: milk is on the horizontal axis and cookies are on the vertical axis.

The current allocation is inefficient. We can see in the diagram that both L and H could be made better off anywhere in the shaded region (both would be on higher ICs).

b) Suggest a reallocation of milk and cookies that will make both Lucie and Hope strictly better off. Illustrate this reallocation in your diagram from part a).

One possible answer: Lucie can give Hope 3 cookies in exchange for 1 glass of milk.

3. Draw a carefully labeled diagram illustrating the contract curve for each of the following cases.

a) $U^A = x+y$; $U^B = x+y$; total x available in the economy = 30 units; total y available in the economy = 15 units.

This is "tricky." $MRS^A = MRS^B = 1$ at *any* allocation. Which tells us that no matter where we are in an Edgeworth Box, each consumers ICs lie on top of each other. i.e., there is never any way to make one person strictly better off without making the other worse off. This means that the contract "curve" is in fact the entire area of the Edgeworth Box.

Think about why this is. Each consumer views the goods as perfect substitutes, no matter how much of each good they have. Thus there are no differences in consumer preferences that will allow mutually beneficial trade.

b) $U^A = xy$; $U^B = xy$; total x available = 20 units; total y available = 20 units.

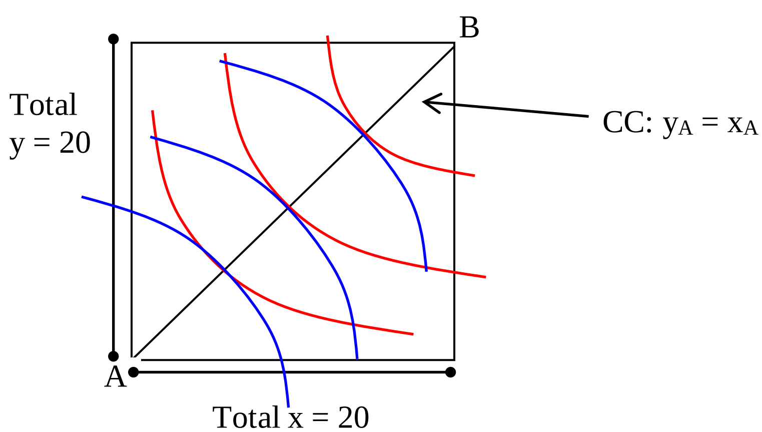
First, let's see whether we can find efficient allocations by equating the MRSs.

$$MRS^A = y_A/x_A = y_B/x_B = MRS^B.$$

Because there are a total of 20 units of x, we know that B's consumption of x is 20 minus A's consumption of x. Similarly, B's consumption of y is 20 minus A's consumption of y. This means that $x_B = 20 - x_A$ & $y_B = 20 - y_A$. So we can rewrite the $MRS^A = MRS^B$ condition from above as:

$$MRS^A = y_A/x_A = (20-y_A)/(20-x_A) = MRS^B.$$
$$\Rightarrow (20-x_A)\, y_A = (20-y_A)\, x_A$$
$$\Rightarrow 20y_A - x_Ay_A = 20x_A - y_Ax_A$$
$$\Rightarrow y_A = x_A$$

Any combination such that y = x for person A is efficient. Note that – because there are equal quantities of x and y – if y = x for A, then y = x for B as well. This tells us that the CC is a straight line with slope 1, that runs from corner to corner in the Edgeworth Box.



c) $U^A = xy$; $U^B = xy$; total x available = 20; total y available = 40.

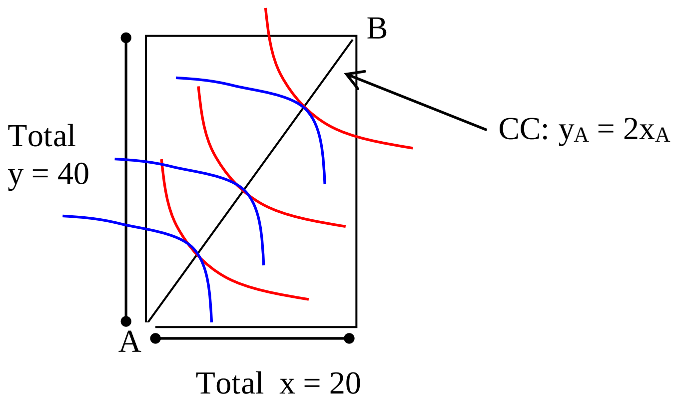
Note that the only difference relative to b) is that we no longer have equal quantities of both the goods. So again, let's begin by equating the MRSs.

$$MRS^A = y_A/x_A = y_B/x_B = MRS^B.$$

Now there are a total of 20 units of x, and 40 units of y. So we have $x_B = 20 - x_A$ & $y_B = 40 - y_A$. Substituting these into the $MRS^A = MRS^B$ condition yields:

$$MRS^A = y_A/x_A = (40-y_A)/(20-x_A) = MRS^B.$$
$$\Rightarrow (20-x_A)\, y_A = (40-y_A)\, x_A$$
$$\Rightarrow y_A = 2x_A$$

Any combination such that y = 2x for person A is efficient. Note that – because there are twice as many units of y as x – if y = 2x for A, then y = 2x for B as well. This tells us that the CC is a straight line with slope 2, that runs from corner to corner in the Edgeworth Box.



d) $U^A = x + y$; $U^B = 2x + y$; total x available = 20; total y available = 20. (harder)

Another "tricky" one. Before you start, note a couple of things about this problem. First, the Edgeworth Box is a square (as total x = total y). Second, $MRS^A = 1$ & $MRS^B = 2$ so B's ICs are always steeper than A's. So you know that this is one of those questions where you will have to draw a diagram and think about the economics, since we can make no progress by trying to equate the MRSs (1 can never equal 2 right?).

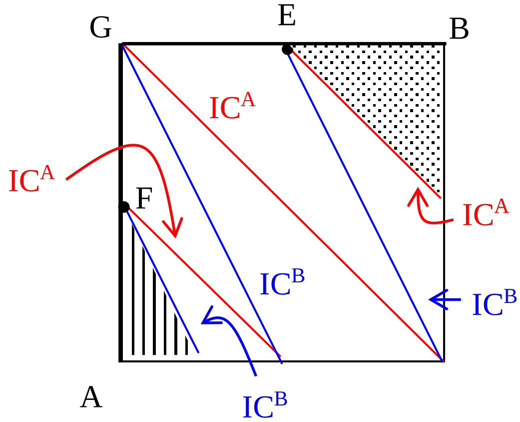
Think about the economics *before* you jump into the diagram. MRS^B is always greater than MRS^A, telling us that B's relative value of good x is always higher than A's. Similarly, A's relative value of y is greater than B's. Efficiency requires us – if its possible – to reallocate the goods in a way that reflects those relative values.

So suppose we consider an allocation where BOTH A and B have positive quantities of both goods (i.e., an allocation that would be in the interior of an EB representing this economy). Given the relative values of each consumer, we know that this is NOT efficient, since we would want to reallocate such that B has more x (she has the greater MRS, the higher relative

value for x) and A has more y (he has the greater relative value for y). This tells us that NO points in the interior of the EB will be efficient, so the CC must lie somewhere on the edges of the box.

Which edges? Think about the economics again for a little while. B has the higher relative value for x and A has the higher relative value for y, so the contract curve can't include an allocation in which A has all the x and B has all the y: it just wouldn't make sense. This means that the CC cannot possibly contain the bottom right corner of the box. What about the top left corner of the EB? This is where A has all the y and B has all the x. It should be clear (make sure it is!) that this surely IS efficient, given the MRSs.

What about other edges of the box? Now it is perhaps time to draw a diagram. When you draw your diagram, keep in mind that the EB should be roughly a square (since there are equal amounts of both goods) and B's ICs will be steeper than A's.



We have already made the case that point G is efficient.

Now consider a point like E. To make A better off than she is at E, we need to move into the dotted region. But this would put B on a lower IC than he is at point E. So E must be efficient (because we cannot make A better off without hurting B), and is part of the CC. This logic holds for *all* points along the top of this Edgeworth Box.

Now consider a point like F. To make B better off than he is at F, we need to move into the striped region. But this would put A on a lower IC than she is at point F. So F must be efficient, and is part of the CC. This logic holds for *all* points along the left edge of this Edgeworth Box. From this we can conclude that the CC runs along the left edge and the top of the Edgeworth Box.

Again, and without trying to labor the point too much, go back to the economic intuition underlying this result. Because $MRS^B > MRS^A$, we know that – on the margin – B values units of x more highly than A (in terms of the quantity of y that each is willing to give up for additional units of x), and A values units of y more highly than B (in terms of the quantity of x that each is willing to give up for additional units of y). This difference in relative values gives us rules for reallocation: always reallocate good x towards B and good y towards A, where such reallocation is feasible. At a point like E, such a reallocation is NOT feasible, since B has only units of x and therefore cannot "swap" y for x with A. The only way, then, that we can make B better off is by taking something away from A and giving it directly to B.

A similar argument holds for points such as F. No reallocation that makes both better off is feasible, since A has no units of good x to offer B in exchange for some of the y that B has. The only way, then, that we can make A better off is by taking something away from B and giving

it directly to A.

e) $U^A = xy^2$ $U^B = xy^2$; total x available = 60; total y available = 20.

You should be getting a sense by now that where we have Cobb-Douglas utility function such as these, the math is a sensible place to start. In these types of questions, begin by finding the MRSs and equating them, and see if this yields a complete answer.

$$MRS^A = y_A/2x_A = y_B/2x_B = MRS^B.$$

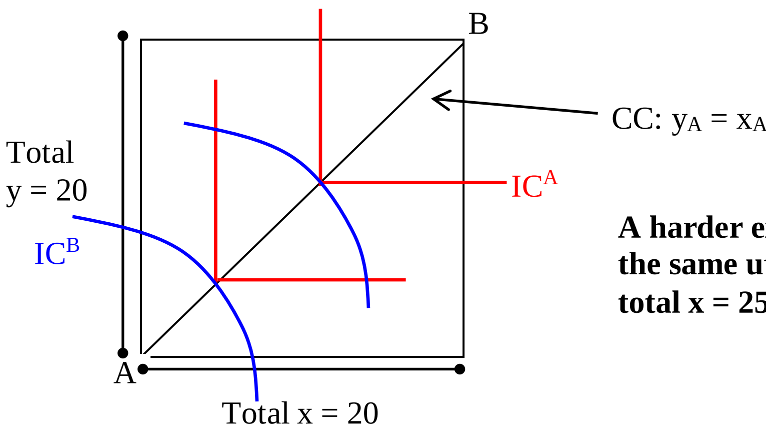
There are a total of 20 units of x, and 40 units of y. So we have $x_B = 60 - x_A$ & $y_B = 20 - y_A$. Substituting these into the $MRS^A = MRS^B$ condition yields:

$$MRS^A = y_A/2x_A = (20-y_A)/2(60-x_A) = MRS^B.$$
$$\Rightarrow (60-x_A)\, y_A = (20-y_A)\, x_A$$
$$\Rightarrow y_A = (1/3)x_A$$

Any combination such that y = (1/3)x for person A is efficient. Note that – because there are one third as many units of y as x – if y = (1/3)x for A, then y = (1/3)x for B as well. This tells us that the CC is a straight line with slope (1/3), that runs from corner to corner in the Edgeworth Box (you can draw this one). Do you notice a pattern emerging here where consumers have *identical* Cobb-Douglas utility functions (look again at parts (b) and (c) and compare the answers there to the answer here)?

f) $U^A = \min\{x,y\}$; $U^B = xy$; total x available = 20; total y available = 20.

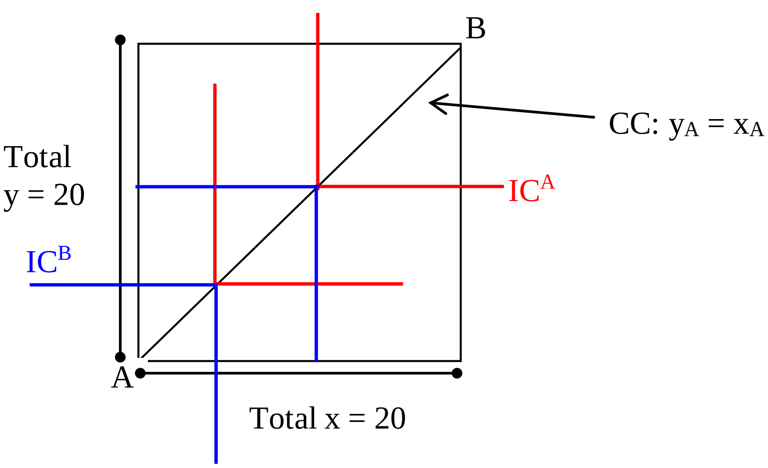
In this case, we know that A's ideal bundle is one such that x = y, which tells us (given B is prepared to make trade-offs between x and y) that $x_A = y_A$ must form part of the contract curve. Because the Edgeworth Box is a square, this in fact is the entire contract curve.



A harder example would be with the same utility functions, but with total x = 25 and total y = 20. <u>Try it</u>.

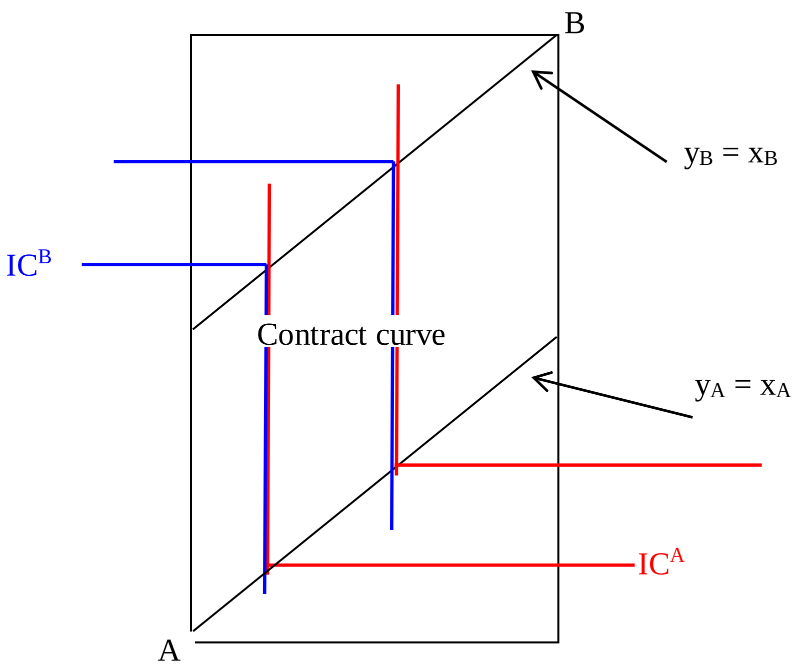
g) $U^A = \min\{x,y\}$; $U^B = \min\{x,y\}$; total x available = 20; total y available = 20.

In this case, both consumers have preferences such that, their ideal bundles contains equal amounts of x and y. In this economy, there are equal amounts of x and y in aggregate, which means that – luckily! – if one consumer has a bundle where x = y, so too will the other consumer. Thus the CC is all those points where x = y for each of them. i.e., it runs corner to corner, as illustrated below.



h) $U^A = \min\{x,y\}$; $U^B = \min\{x,y\}$; total x available = 20; total y available = 40.

This looks awfully similar to g). The only difference is that we no longer have equal quantities of both goods. We have – in some sense – "excess" y. Both consumers would ideally like to have x = y, but because aggregate x does not equal aggregate y, this won't be possible. Either A or B (or both) will end up with some "extra" y in their consumption bundles, no matter what. Note that this doesn't make them worse off: given the utility functions as specified, units of y in excess of units of x don't increase OR decrease their utility. Think about this, and about the solution to the last problem, and convince yourself that you understand why the CC is therefore as drawn below.



The CC is anywhere where $y_B = x_B$, anywhere where $y_A = x_A$, *and* at any point in the region between these two lines.

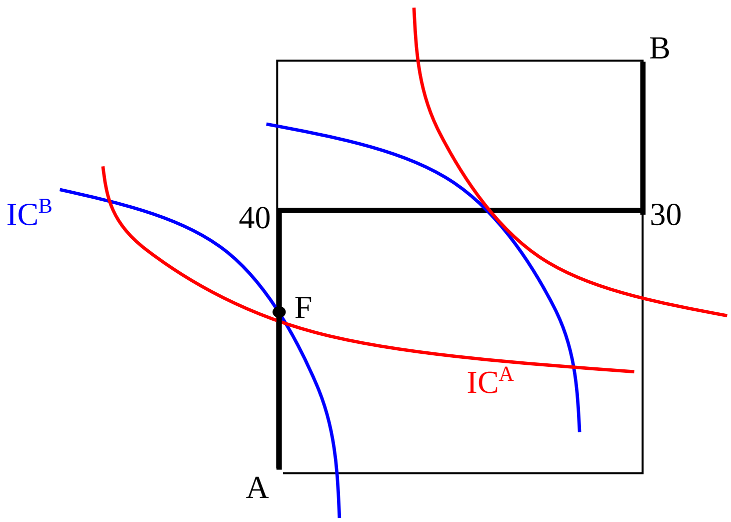
i) $U^A = 3x + \ln y$; $U^B = 4x + \ln y$ total x available = 40; total y available = 70.

These utility functions are referred to as "quasi-linear" utility functions (the utility function is linear in x, but not in y, so its is "kind of" or "quasi"-linear). They give rise to strictly convex ICs, but you need to be careful, because – unlike Cobb-Douglas utility functions – the ICs in this case do hit (at least one of) the axes (the y axis this case). This means that you need to be concerned about the sides of the Edgeworth Box, and whether they are part of the CC.

As a first step, see if equating the MRSs helps get you part way to the answer.

$$MRS^A = 3y_A = 4y_B = MRS^B.$$
$$\Rightarrow 3y_A = 4(70 - y_A)$$
$$\Rightarrow 3y_A = 280 - 4y_A$$
$$\Rightarrow 7y_A = 280$$
$$\Rightarrow y_A = 40 \ \& \ y_B = 30 \text{ is part of the CC}$$

This tells us that at least part of the CC is the horizontal line drawn below. At each point along this line, the MRSs are equal and there are no reallocation such that at least one consumer is better off without the other being worse off .



But this isn't all there is to the CC. Take a point like F, where $y_A = 20$ and $y_B = 50$. At that point, $MRS^A = 60$ and $MRS^B = 200$, meaning that A's IC is flatter than B's. Given this, there is no attainable region in which both can be made better off, so this point is efficient.

In other words, at a point like F, B's relative value of x is higher than A's, and A's relative value of y is higher than B's. If it were possible, we would therefore like to reallocate the goods such that B has more x and less y, and A has more y and less x. But at point F, B already has all the x available in the economy, so such a swap is not possible.

This same logic holds in the region where $x_B = 0$; $y_A > 40$, $y_B < 30$ (check this for yourself, however). Thus the CC begins at A's origin, travels up the left edge of the box to point where $y_A = 40$, then continues horizontally through the interior of the EB. Finally, it runs up the right edge to B's origin.

4. Consider the following information about a pure exchange economy in which there are two goods and two consumers:

Consumer A: $U^A = (20 - 0.125x_A)x_A + y_A$; $w^A{}_x = 80$; $w^A{}_y = 50$.
Consumer B: $U^B = (20 - 0.1x_B)x_B + y_B$; $w^B{}_x = 10$; $w^B{}_y = 500$.

a) Solve for each consumer's demand function for good x.

Recall that a demand function tells us how many units of a good (say x) that a consumer will buy, as a function of (relative) price, where consumers make consumption choices in order to maximize utility.

Utility maximization here:
Consumer A:

$$\mathbf{MRS^A = 20 - 0.25x_A = p_x/p_y}$$
$$\mathbf{\Rightarrow x_A = 80 - 4(p_x/p_y)}$$

This is A's demand for x. Similarly, we can find B's demand function, which is:

$$\mathbf{x_B = 100 - 5(p_x/p_y)}$$

Note that – because the utility functions are quasi-linear, the demand for x is not a function of y, since the MRS is not a function of y. Neither therefore is the demand for x a function of income (the endowment). This means we need to be a little careful: suppose that prices were such that each consumer wished to purchase a bundle that they in fact cannot afford? Because the demand function does not incorporate income explicitly, this (impossible outcome) may happen. We need to also remember that the consumer is constrained in their choices by their income, so always check that you are solving for a quantity that is in fact affordable (you needed to do this in question 1d) and e), right?). Don't worry however: I won't try and trip you up with such corner solutions on exams!

b) Solve for the equilibrium price ratio in this economy.

Aggregate demand is $x_A + x_B = \{80 - 4(p_x/p_y)\} + \{100 - 5(p_x/p_y)\} = 180 - 9(p_x/p_y)$.
There are 90 units of good x in total in the economy, so the price ratio that equates supply and demand is 10. That is, each unit of x trades for 10 units of y.

c) How much of each good do A and B buy, sell, and consume in the equilibrium?

Given the price ratio, we know that A consumes a total of 40 units of good x. Given that she is endowed with 80 units, she must be selling 40 of her 80 units to consumer B. For each unit of x she sells, she receives 10 units of y, therefore she is paid a total of 400 units of good y. She then consumes 450: the 400 she receives from A plus the 50 in her endowment.

We know that B buys 40 units of good x from A. He therefore consumes 50 units: the 40 he buys plus the 10 he is endowed with. In order to pay for those 40 units of x, he must give A 400 units of y, meaning that he consumes only 100 of the 500 he was endowed with.

5. Consider the following information about a pure exchange economy in which there are two goods and two consumers:

Consumer A: $U^A = x_A^2 y_A$; $w^A{}_x = 5$; $w^A{}_y = 15$.
Consumer B: $U^B = x_B^2 y_B$; $w^B{}_x = 15$; $w^B{}_y = 5$.

a) Solve for the equilibrium price ratio and the equilibrium consumption choices for each consumer.

The trick here is to try and follow the same steps as in question 2 above. You are simply solving for the demands and setting demand equal to supply. What makes this more complicated is that the MRS is now a function of BOTH goods consumed. But if you follow the same steps, you will get the right answer. We know we have to 1. solve the consumer's utility maximization problem to derive their demands and 2. solve for prices that equate supply and demand.

1. Utility maximization:

For consumer A we have :

$$MRS^A = \frac{2y_A}{x_A} = \frac{p_x}{p_y} \Rightarrow y_A = \frac{1}{2}\frac{p_x}{p_y}x_A$$

$$BL^A : \frac{p_x}{p_y}x_A + y_A = \frac{p_x}{p_y}w_x^A + w_y^A$$

$$\Rightarrow \frac{p_x}{p_y}x_A + \frac{1}{2}\frac{p_x}{p_y}x_A = \frac{p_x}{p_y}5 + 15$$

$$\Rightarrow \frac{3}{2}\frac{p_x}{p_y}x_A = \frac{p_x}{p_y}5 + 15$$

$$\Rightarrow x_A = \frac{10}{3} + \frac{10}{p_x/p_y}$$

This is A's demand for x.

Similarly, for consumer B we have:

$$MRS^B = \frac{2y_B}{x_B} = \frac{p_x}{p_y} \Rightarrow y_B = \frac{1}{2}\frac{p_x}{p_y}x_B$$

$$BL^B : \frac{p_x}{p_y}x_B + y_B = \frac{p_x}{p_y}w_x^B + w_y^B$$

$$\Rightarrow \frac{p_x}{p_y}x_B + \frac{1}{2}\frac{p_x}{p_y}x_B = \frac{p_x}{p_y}15 + 5$$

$$\Rightarrow \frac{3}{2}\frac{p_x}{p_y}x_B = \frac{p_x}{p_y}15 + 5$$

$$\Rightarrow x_B = 10 + \frac{1}{3}\frac{10}{p_x/p_y}$$

Aggregate demand is:

$$x_A + x_B = \left(\frac{10}{3} + \frac{10}{p_x/p_y}\right) + \left(10 + \frac{1}{3}\frac{10}{p_x/p_y}\right)$$

$$= \frac{40}{3} + \frac{1}{3}\frac{40}{p_x/p_y}$$

Aggregate supply of good x is 20, therefore the equilibrium price ratio is where:

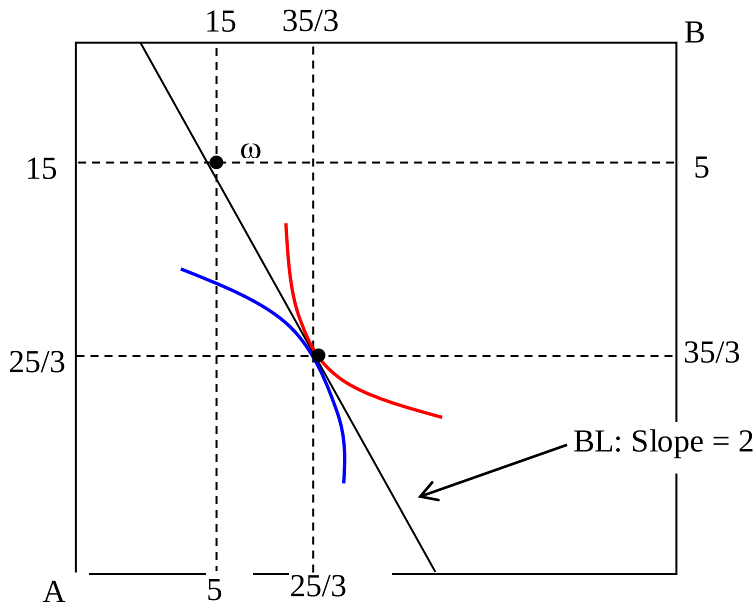
$$20 = \frac{40}{3} \Rightarrow 60 = 40 + \frac{40}{p_x/p_y}$$

$$\Rightarrow 20 = \frac{40}{p_x/p_y}$$

$$\Rightarrow p_x/p_y = 2$$

Given this price ratio, we know that A consumes 25/3 units of good x and B consumes 35/3 units of good x. From A's and B's conditions we can solve for their consumption of y. This will be 25/3 units of y for A and 35/3 units of y for B.

b) Draw a carefully labeled Edgeworth Box diagram illustrating this equilibrium.



6. Now suppose that you have the same information as in question 3, but now consumer B has the utility function $U^B = x_B + y_B$.

a) Demonstrate that the equilibrium price ratio from question 3 is *not* the price ratio for this economy.

Now B has straight line ICs with MRS = 1. If the price ratio is 2, then B will want to consume all y and no x. We already know that – at this price ratio – A wants to consume x = 25/3. Thus supply and demand for x are not equal at a price ratio of 2 (excess supply of x), given the new utility function for B. The equilibrium price ratio must be less than 2.

b) Explain why the equilibrium price ratio must equal 1 in this economy.

The argument above holds for all price ratios greater than 1, since B will always want to consume no x if the price of x is greater than the price of y. So B will want to sell all 15 of his units of good x. Recall that A's demand curve is:

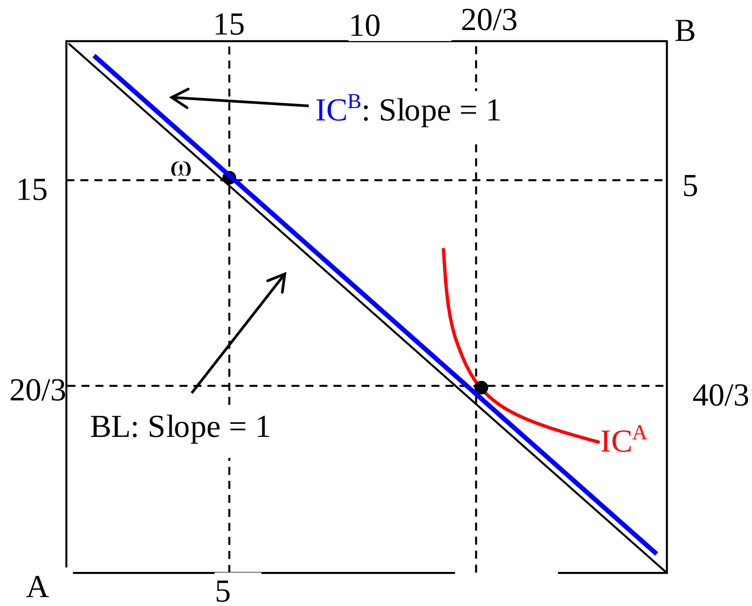
$$x_A = \frac{10}{3} + \frac{10}{p_x/p_y},$$

and note that if $p_x/p_y > 1$**, then** $x_A < 40/3$ **(~13.33). Thus for all price ratios greater than 1, aggregate supply of good x is greater than aggregate demand for good x.**

Now suppose the price ratio is less than 1. In this case, B will want to consume only x (and no y) and hence will not be prepared to sell any x to A. What will A's demand be at price ratios less than 1? From the demand function we can see that if $p_x/p_y < 1$, then $x_A > 40/3$ (~13.33). So now demand for x exceeds supply of x, putting upward pressure on the relative price of x.

The only possibility is therefore that $p_x/p_y = 1$. Given this, A's (gross) demand for x will be 40/3 units, meaning that she wishes to buy 25/3 units in total from B. B is indifferent between selling and not units of x given this price ratio, and we assume that he will therefore be OK selling to A. So supply equals demand at the price ratio of 1.

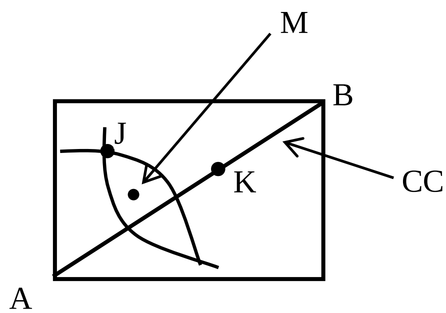
c) Draw a carefully labeled Edgeworth Box diagram illustrating the equilibrium in this economy.



7. Are the following statements true or false? Explain your reasoning carefully and illustrate in each case using an Edgeworth Box diagram.

a) Every movement from a Pareto inefficient point to a Pareto efficient point is a Pareto improvement.

False. Moving from J to K is a move from an inefficient point to an efficient one, but B is made worse off, so its not a Pareto improvement.



b) Every Pareto improvement involves moving from a Pareto inefficient point to a Pareto efficient point.

False. Moving from point J to a point like M would put both consumers on a higher IC (and hence is a Pareto improvement), but is not efficient (not on the CC: there are further Pareto improvements to be had).