RAMAKRISHNA MISSION RESIDENTIAL COLLEGE (Autonomous)

Narendrapur, Kolkata - 700103

SYLLABUS

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B.Sc. THREE-YEAR HONOURS
DEGREE COURSE ON
MATHEMATICS

SEMESTER SYSTEM

MATHEMATICS July, 2012

MATHEMATICS HONOURS

SEMESTER-WISE DISTRIBUTION

Semester – 1	Paper I	and	Paper II
Semester – 2	Paper III	and	Paper IV
Semester – 3	Paper V	and	Paper VI
Semester – 4	Paper VII	and	Paper VIII
Semester – 5	Paper IX	and	Paper X
	Paper XI	and	Paper XII
Semester – 6	Paper XIII	and	Paper XIV
	Paper XV	and	Paper XVI

MATHEMATICS HONOURS

DISTRIBUTION OF MARKS

Semester – 1

PAPER I: Group A: Classical Algebra (30 marks)
Group B: Modern Algebra I (20 marks)

PAPER II: Group A: Analytical Geometry of 2 Dimensions (20 marks) **Group B:** Analytical Geometry of 3 Dimensions I (15 marks)

Group C: Real Analysis I (15 marks)

Semester – 2

PAPER III: Group A: Real Analysis I I (25 marks)

Group B: Evaluation of Integrals (10 marks)

Group C: Vector Algebra (15 marks)

PAPER IV: Group A: Modern Algebra II (20 marks)

Group B: Linear Algebra I (15 marks)

Group C: Analytical Geometry of 3 Dimension II (15 marks)

Semester - 3

PAPER V: Group A: Linear Algebra II & Modern Algebra III (15 marks) **Group B:** Linear Programming and Game Theory (35 marks)

PAPER VI : Group A : Real Analysis III (15 marks)

Group B: Differential Equations I (35 marks)

Semester - 4

PAPER VII : Group A : Real-valued Functions of Several Real variables(30marks)

Group B: Application of Calculus (20 marks)

PAPER VIII : Group A : Vector Calculus (25 marks)

Group B: Analytical Dynamics of A Particle I (25 marks)

Semester - 5

PAPER IX : Group A : Real Analysis IV(50 marks)

PAPER X : Group A : Linear Algebra III (20 marks)

Group B: Tensor Calculus (15 marks)

Group C: Differential Equation II (15 marks)

PAPER XI : Group A : Analytical Statics (30 marks)

Group B: Analytical Dynamics of A Particle II(20 marks)

PAPER XII : Group A : Hydrostatics (25 marks)

Group B: Rigid Dynamics (25 marks)

Semester – 6

PAPER XIII : Group A : Real Analysis V (20 marks)

Group B: Metric Space (15 marks)Group C: Complex Analysis (15 marks)

PAPER XIV : Group A : Probability (30 marks)

Group B: Statistics (20 marks)

PAPER XV : Group A : Numerical Analysis (25 marks)

Group B: Computer Programming (25 marks)

PAPER XVI : **Practical** (50 marks)

Problem: 40 Sessional Work: 5

Viva: 5

SEMESTER – 1

Paper I

Group A (30 marks)

Classical Algebra

1. Statements of well ordering principle, first principle of mathematical induction, second principle of mathematical induction. Proofs of some simple mathematical results by induction. Divisibility of integers. The division algorithm (a = gb + r, b ≠ 0, 0 ≤ r < b). The greatest common divisor (g.c.d.) of two integers a and b. [This number is denoted by the symbol (a,b)]. Existence and uniqueness of (a,b). Relatively prime integers. The equation ax + by = c has integral solution iff (a,b) divides c. (a, b, c are integers).

Prime integers. Euclid's first theorem: If some prime p divides ab, then p divides either a or b.

Euclid's second theorem: There are infinitely many prime integers.

Unique factorization theorem. Congruences, Linear Congruences.

Statement of Chinese Remainder Theorem and simple problems. Theorem of Fermat. Multiplicative function Ø(n). [15]

- Complex Numbers: De-Moivre's Theorem and its applications, Exponential, Sine, Cosine and Logarithm of a complex number. Definition of a ^z (a≠0). Inverse circular and Hyperbolic functions. [8]
- 3. Polynomials with real co-efficients: Fundamental theorem of Classical Algebra (statement only). The n-th degree polynomial equation has exactly n roots. Nature of roots of an equation (surd or complex roots occur in pairs). Statements of Descartes' rule of signs and of Sturm's Theorem and their applications. Multiple roots. Relation between roots and coefficients. Symmetric functions of roots. Transformation of equations.
- **4.** Polynomial equations with real co-efficients: Reciprocal equations. Cardan's method of solving a cubic equation. Ferrari's method of solving a biquadratic equation. Binomial equation. Special roots. [7]
- 5. Statements of Inequalities AM ≥ GM ≥ HM and their generalizations: the theorem of weighted means and m-th. Power theorem, Cauchy's inequality and their direct applications. [8]

Group B (20 marks)

Modern Algebra I

1. Revision of basic set theory, Binary relation, Equivalence relation and partition, fundamental theorem of equivalence relation, congruence classes, of integers and \square .

Mapping: Injective, Surjective and bijective maps, invertability and inverse of a mapping, composition of maps, binary operations, groupoid.

Group Theory: Semi group, Group, Abelian Group. Various examples of groups(Klein's 4 Group, Groups of congruence classes.), elementary properties, conditions for a semi-group to be a group, integral power of elements and order of an element in a group. Permutation groups, cycle, transposition, every permutation can be expressed as a product of disjoint cycles (Statement only), even and odd permutations, symmetric group, alternating group.

Sub Group: Necessary and sufficient conditions for a nonempty subset of a group to be a subgroup. Intersection and union of sub groups. Necessary and sufficient condition for union of two subgroups to be a subgroups. [10]

Question Pattern

Paper – I (50 Marks)

Group A (30 marks)
<u>Classical Algebra</u>

There will be 5 questions, one from each topic, each of 6 marks. Each such question will have further parts with alternatives.

Group B (20 marks) Modern Algebra

Four questions each carrying 5 marks are to be answered out of 7 questions. Each such question may have further parts.

Paper II

Group A (20 marks) **Analytical Geometry of Two Dimensions**

- 1. (a) Transformation of Rectangular axes: Translation, Rotation and their combinations. Theory of Invariants. [2]
 - (b) General Equation of second degree in two variables: Reduction into canonical form. Classification of conics. Length and position of the axes. [2]
- Pair of straight lines: Condition that the general equation of second degree in two variables may represent two straight lines. Point of intersection of two intersecting straight lines. Angle between two lines given by $ax^2 + 2hxy + by^2 = 0$. Angle bisector. Equation of two lines joining the origin to the points in which a line meets a conic. [8]
- **3.** Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equations of tangent, normal, chord of contact. [5]
- 4. Circle, Parabola, Ellipse and Hyperbola: Equations of pair of tangents from an external point, chord of contact, poles and polars, conjugate points and conjugate lines. [4]

Note: Euclid's Axiom and its Consequences.

Group B (15 marks)

Analytical Geometry of Three Dimensions I

- 1. Rectangular Cartesian co-ordinates in space. Halves and Octants. Concept of a geometric vector (directed line segment). Projection of a vector on a co-ordinate axis. Inclination of a vector with an axis. Co-ordinates of a vector. Direction cosines of a vector. Distance between two points. Division of a directed line segment in a given ratio. [4]
- 2. Equation of Plane: General form, Intercept and Normal form. The sides of a plane. Signed distance of a point from a plane. Equation of a plane passing through the intersection of two planes.

 Angle between two intersecting planes. Bisectors of angles between two intersecting planes. Parallelism and perpendicularity of two planes. [8]
- 3. Straight lines in space: Equation (Symmetric & Parametric form). Direction ratio and Direction cosines. Canonical equation of the line of intersection of two intersecting planes. Angle between two lines. Distance of a point from a line. Condition of coplanarity of two lines. Equations of skew-lines. Shortest distance between two skew lines. [10]

Group C (15 marks) Real Analysis I

1. Real number system:

- (a) Intuitive idea of numbers. Mathematical operations revisited with their properties (closure, commutative, associative, identity, inverse, distributive). Sets and functions definition and properties (union, intersection, complementation, injection, surjection, bijection). [3]
- (b) Field Axioms. Concept of ordered field. Bounded set, L.U.B. (supremum) and G.L.B. (infimum) of a set. Properties of L.U.B. and G.L.B. of sum of two sets and scalar multiple of a set. Least upper bound axiom or completeness axiom. Characterization of IR as a complete ordered field. Definition of an Archimedean ordered field. Archimedean property of IR. Q is Archimedean ordered field but not ordered complete. Linear continuum. [6]

2. Sets in IR:

(a)Intervals. [1]

- (b) Neighbourhood of a point, Interior point, Open set. Union, intersection of open sets. Every open set can be expressed as disjoint union of open intervals(statement only). [2]
- (c) Limit point and isolated point of a set. Criteria for L.U.B. and G.L.B. of a bounded set to be limit point of the set. Bolzano-Weierstrass theorem on limit point. Definition of derived set. Derived set of a bounded set A is contained in the closed interval [inf A, supA]. Closed set. Complement of open set and closed set. Union and intersection of closed sets as a consequence. No nonempty proper subset of IR is both open and closed. [3]
 - (d) Dense set in IR as a set having non-empty intersection with every open interval. Q and IR-Q are dense in IR. [2]
- 3. Countability of sets: Countability (finite and infinite) and uncountability of a set. Subset of a countable set is countable. Every infinite set has a countably infinite subset. Cartesian product of two countable sets is countable. Q is countable. Non-trivial intervals are uncountable. IR is uncountable. [4]

Question Pattern

Paper – II (50 Marks)

Group A (20 marks)

Analytical Geometry of Two Dimensions

There will be 4 questions, one from each topic, each of 5 marks. Each such question may have further parts with alternatives.

Group B (15 marks)

Analytical Geometry of Three Dimensions I

There will be 3 questions, one from each topic, each of 5 marks. Each such question may have further parts with alternatives.

Group C (15 marks)

Real Analysis I

One question of 15 marks is to be answered out of 2 such questions. Each such question may have further parts covering the entire syllabus.

SEMESTER – 2

Paper III

Group A (25 marks)

Real Analysis II

1. Sequences of real numbers :

- (a) Definition of a sequence as function from N to R. Bounded sequence. Convergence (formalization of the concept of limit as an operation in R) and non-convergence. Examples. Every convergent sequence is bounded and limit is unique. Algebra of limits. [4]
- (b) Relation between the limit point of a set and the limit of a convergent sequence of distinct elements. Monotone sequences and their convergence. Sandwich rule. Nested interval theorem. Limit of some important sequences:

$$\{n^{\frac{1}{n}}\}, \{x^n\}_n, \{x^{\frac{1}{n}}\}_n, \{x_n\}_n \text{ with } \frac{x_n+1}{x_n} \to l \text{ and}$$

$$|l| < 1, \{(1+\frac{1}{n})^n\}, \left\{1+\frac{1}{1!}+\frac{1}{2!}+\dots+\frac{1}{n!}\right\}_n, \{a^{x_n}\}_n \text{ (a > 0)}$$
Cauchy's first and second limit theorems. [7]

(c) Subsequence. Subsequential limits. Lim sup (upper limit) and lim inf (lower limit) of a sequence using inequalities. Alternative definitions of lim sup and lim inf of a sequence $\{x^n\}_n$ using L.U.B. and G.L.B. of the set containing all the subsequential limits or by the properties of the set $\{x_n, x_{n+1},\}$ (Equivalence between these definitions are assumed). A bounded sequence $\{x_n\}$ is convergent if lim sup $\{x_n\}$ =lim inf $\{x_n\}$ (statement only). Every sequence has a monotone subsequence. Bolzano-Weierstrass theorem. Cauchy's general principle of convergence [5]

2. Real-valued functions of a real variable :

(a) Limit of a function at a point (the point must be a limit point of the domain set of the function). Sequential criteria for the existence of finite and infinite limit of a function at a point. Algebra of limits. Sandwich rule. Important limits like

$$\frac{\sin x}{x}$$
, $\frac{\log(1+x)}{x}$, $\frac{a^x-1}{x}$ $(a>0)$ as $x\to 0$

(b) Continuity of a function at a point. Continuity of a function on an interval and at an isolated point. Familiarity with the figures of some well known functions :

$$y = x^a \left(a = 2, 3, \frac{1}{2}, -1 \right), |x|, \sin x, \cos x, \tan x, \log x, e^x$$

Algebra of continuous functions as a consequence of algebra of limits. Continuity of composite functions. Examples of continuous functions. Continuity of a function at a point does not necessarily imply the continuity in some neighbourhood of that point. [4]

- (c) Bounded functions. Neighbourhood properties of continuous functions regarding boundedness and maintenance of same sign. Continuous function on [a,b] is bounded and attains its bounds. Intermediate value theorem. [4]
- (d) Discontinuity of function, type of discontinuity. Step function. Piecewise continuity. Monotone function. Monotone function can have only jump discontinuity. Set of points of discontinuity of a monotone function is at most countable. Monotone bijective function from an interval to an interval is continuous and its inverse is also continuous.
- (e) Definition of uniform continuity and examples. Lipschitz condition and uniform continuity. Functions continuous on a closed and bounded interval is uniformly continuous. A necessary and sufficient condition under which a continuous function on a bounded open interval *I* will be uniformly continuous on *I*. A sufficient condition under which a continuous function on an unbounded open interval *I* will be uniformly continuous on *I* (statement only).

[5]

Group B (10 marks)

Evaluation of Integrals

Evaluation of Integrals: Indefinite and suitable corresponding definite integrals

for the functions
$$\frac{1}{(a+b\cos x)^n}$$
, $\frac{l\cos x + m\sin x}{p\cos x + q\sin x}$, $\frac{1}{(x^2+a^2)^n}$, $\cos^m x$, $\sin^n x$,

etc. where l, m, p, q, n are integers. Simple problems on definite integral as the limit of a sum, elementary idea of double and triple integrals.

[5]

Group C (15 marks)

Vector Algebra

Vector Algebra: Vector (directed line segment) Equality of two free vectors. Addition of Vectors. Multiplication by a Scalar.

Position vector, Point of division, Conditions of collinearity of three points and co-planarity of four points.

Rectangular components of a vector in two and three dimensions.

Product of two or more vectors. Scalar and vector products, scalar triple products and Vector triple products. Product of four vectors.

Direct application of Vector Algebra in (i) Geometrical and Trigonometrical problems (ii) Work done by a force, Moment of a force about a point.

Vector equations of straight lines and planes. Volume of a tetrahedron. Shortest distance between two skew lines. [15]

Question Pattern

Paper - III (50 marks)

Group - A (Real Analysis - II) - 25 marks:

One compulsory question of 5 marks with alternatives is to be answered. **Two** questions, **each of 10 marks**, are to be answered out of **four** such questions. Each such question may have further parts.

Group - B (Evaluation of Integrals) - 10 marks:

Two questions, each of 5 marks, are to be answered out of **four** questions covering the whole syllabus. Each such question may have further parts.

Group - C (Vector Algebra) - 15 marks:

Three questions, **each of 5 marks**, are to be answered out of **five** questions covering the whole syllabus. Each such question may have further parts.

Paper IV

Group A (20 marks)

Modern Algebra II

- 1. Cyclic Group, generator, subgroups of a cyclic group. Necessary and sufficient condition for a finite group to be cyclic; cosets and lagranges's theorem on finite group; counter exmple to establish the failure of the converse.
- **2.** Normal subgroup of a group, intersection and union of normal subgroups, quotient group. Homomorphism and isomorphism of groups, kernel of a homormorphism, first isomorphism theorem, classification of finite and infinite cyclic groups. Cayley's theorem, homomorphic image of a group.
- **3.** Rings and fields. Elementary properties, unitary and commutative rings, divisor of zero, integral domain. Every field is an integral domain, finite integral domain is a field.

Group B (15 marks)

Linear Algebra I

- 1. Matrices of real and complex numbers; algebra of matrices, symmetric and skew symmetric matrices, Hermetian and skew-Hermetian matrices, orthogonal and unitary matrices.
- 2. Determinants; definition, basic properties, minors and cofactors. Laplace expansion, Vandermonde's determinant. Symmetric and skew symmetric determinants (no proof of theorems required, problems on determinants up to order 4).

Adjoint of a square matrix. For a square matrix A, A.adj A= adj A.A= (det A) I_n.

Invertible matrix, non singularity. Inverse of an orthogonal matrix. Jacobi's Theorem (statement only) and it's application.

- 3. Elementary operations on matrices. Echelon matrix. Determination of a rank of a matrix (statement and relevant results only). Triangular factorization A= LDU, Elementary matrices, Normal form, Evaluation of determinant by Gaussian elimination.
- 4. Congruence of matrices: statement and application of relevant results, Normal form of a matrix under congruence.

Group C (15 marks)

Analytical Geometry of 3 Dimensions II

- **1.** (a) Sphere: General Equation. Circle, Sphere through the intersection of two spheres. Radical Plane, Tangent, Normal. [3]
 - (b) Cone: Right circular cone. General homogeneous second degree equation. Section of cone by a plane as a conic and as a pair of lines. Condition for three perpendicular generators. Reciprocal cone. [5]
 - (c) Cylinder: Generators parallel to either of the axes, general form of equation. Right-circular cylinder.[2]
 - (d) Ellipsoid, Hyperboloid, Paraboloid :Canonical equations only.[1]
- **2.**Tangent planes, Normal, Enveloping cone. [5]
- **3.** Surface of Revolution (about axes of reference only). Ruled surface. Generating lines of hyperboloid of one sheet and hyperbolic paraboloid. [10]
- **4.** Transformation of rectangular axes by translation, rotation and their combinations. [2]
- **5.** Knowledge of Cylindrical, Polar and Spherical polar co-ordinates, their relations (no deductions required). [2]

Question Pattern

Paper - IV (50 marks)

Group - A (Modern Algebra II) - 20 marks

Two questions, each of 10 marks, are to be answered out of **four** questions covering the whole syllabus. Each such question may have further parts.

Group - B (Linear Algebra I) -15marks

Three questions, each of 5 marks, are to be answered out of five questions covering the whole syllabus. Each such question may have further parts.

Group - C (Analytical Geometry of 3Dimensions II) -15marks

Three questions, **each of 5 marks**, are to be answered out of **seven** questions covering the whole syllabus. Each such question may have further parts.

<u>SEMESTER – 3</u>

Paper V

Group A (15 marks)

Modern Algebra III and Linear Algebra II

- 1. Characteristic of a ring and of an integral domain. Sub ring and sub field.
- **2.** Ideals and homomorphisms of rings (statement of relevant theorems), prime ideals, maximal ideal—definition, example and related theorems.
- 3. Vector/ Linear space: Definition and examples, subspace, union and intersection of subspaces, linear sum of two subspaces. Linear combination independence and dependence of vectors, linear span, generator of a vector space, finite dimensional real vector space, basis. Deletion theorem, extension theorem, replacement theorem. Dimension, extraction of basis. Vector space over a finite field.

Group B (35 marks)

Linear Programming and Game Theory

- 1. Definition of L.P.P. Formation of L.P.P. from daily life involving inequations. Graphical solution of L.P.P. Basic solutions and Basic Feasible Solution (BFS) with reference to L.P.P. Matrix formulation of L.P.P. Degenerate and Non-degenerate B.F.S. [8]
- 2. Hyperplane, Convex set, Cone, extreme points, convex hull and convex polyhedron. Supporting and Separating hyperplane. The collection of all feasible solutions of an L.P.P. constitutes a convex set. The extreme points of the convex set of feasible solutions correspond to its B.F.S. and conversely. The objective function has its optimal value at an extreme point of the convex polyhedron generated by the set of feasible solutions. (the convex polyhedron may also be unbounded). In the absence of degeneracy, if the L.P.P. admits of an optimal solution then at least one B.F.S. must be optimal. Reduction of a F.S. to a B.F.S.
- **3.** Slack and surplus variables. Standard from of L.P.P. theory of simplex method. Feasibility and optimality conditions [6]
- **4.** The algorithm. Two phase method. Degeneracy in L.P.P. and its resolution. [6]

- 5. Duality theory: The dual of the dual is the primal. Relation between the objective values of dual and the primal problems. Relation between their optimal values. Complementary slackness, Duality and simplex method and their applications. [6]
- **6.** Transportation and Assignment problems. Mathematical justification for optimality criterion. Hungarian method. Traveling Salesman problem. [8]
- 7. Concept of Game problem. Rectangular games. Pure strategy and Mixed strategy. Saddle point and its existence. Optimal strategy and value of the game. Necessary and sufficient condition for a given strategy to be optimal in a game. Concept of Dominance. Fundamental Theorem of Rectangular games. Algebraic method. Graphical method and Dominance method of solving Rectangular games. Inter-relation between the theory of Games and L.P.P. [10]

Question Pattern

Paper – V (50 Marks)

Group A (15 marks) Modern Algebra III & Linear Algebra II

One question of 7 marks is to be answered out of 2 such question covering Topic 1 & 2.

Two questions of 4 marks each are to be answered out of 4 questions covering Topic 3.Each question may have further subdivisions.

Group B (35 marks) Linear Programming and Game Theory

Five questions each carrying 7 marks are to be answered out of 9 questions covering the entire syllabus. Each such question may have further parts.

Paper VI

Group A (15 marks)

Real Analysis III

1. Infinite Series of real numbers :

a) Convergence, Cauchy's criterion of convergence.

[1]

- b) Series of non-negative real numbers: Tests of convergence Cauchy's condensation test. Comparison test (ordinary form and upper limit and lower limit criteria), Kummer's test. Statements and applications of:
 Abel Pringsheim's Test, Ratio Test, Root test, Raabe's test,
 Bertrand's test, Logarithmic test and Gauss's test.
- c) Series of arbitrary terms : Absolute and conditional convergence
- **d)** Alternating series: Leibnitz test (proof needed).
- e) Non-absolute convergence: Abel's and Dirichlet's test (statements and applicatins). Riemann's rearrangement theorem (statement only) and rearrangement of absolutely convergent series (statement only).

[3]

2. Derivatives of real –valued functions of a real variable:

a) Definition of derivability. Meaning of sign of derivative. Chain rule.

[2]

b) Successive derivative: Leibnitz' theorem

- [1]
- c) Theorems on derivatives: Darboux theorem, Rolle's theorem, Mean value theorems of Lagrange and Cauchy as an application of Rolle's theorem. Taylor's theorem on closed and bounded interval with Lagrange's and Cauchy's form of remainder deduced from Lagrange's and Cauchy's mean value theorem respectively. Maclaurin's theorem as a consequence of Taylor's theorem. Statement of Maclaurin's

Theorem on infinite series expansion. Expansion of e^x , $\log(1+x),(1+x)^m$, $\sin x$, $\cos x$ with their range of validity.

[10]

d) Statement of L' Hospital's rule and its consequences. Point of local extremum (maximum, minimum) of a function in an interval. Sufficient condition for the existence of a local maximum/minimum of a function at a point (statement only). Determination of local extremum using first order derivative. Application of the principle of maximum/minimum in geometrical problems. [4]

Group B (35 marks)

Differential Equation I

- Significance of ordinary differential equation. Geometrical and physical consideration. Formation of differential equation by elimination of arbitrary constant. Meaning of the solution of ordinary differential equation. Concept of linear and non-linear differential equations. [2]
- 2. Equations of first order and first degree: Statement of existence theorem. Separable, Homogeneous and Exact equation. Condition of exactness, Integrating factor. Rules of finding integrating factor, (statement of relevant results only). [5]
- **3.** First order linear equations: Integrating factor (Statement of relevant results only). Equations reducible to first order linear equations. [2]
- **4.** Equations of first order but not of first degree. Clairaut's equation. Singular solution. [3]
- **5.** Applications: Geometric applications, Orthogonal trajectories. [2]
- 6. Higher order linear equations with constant coefficients: Complementary function, Particular Integral. Method of undetermined coefficients, Symbolic operator D. Method of variation of parameters. Exact Equation. Euler's homogeneous equation and reduction to an equation of constant coefficients.
- 7. Second order linear equations with variable coefficients: Reduction of order when one solution of $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = F(x)$ the homogeneous part is known. Complete solution. Method of variation of parameters. Reduction to Normal form. Change of independent variable. Operational Factors. [10]
- **8.** Simple eigenvalue problems. [2]
- **9.** Simultaneous linear differential equations. Total differential equation : Condition of integrability. [3]
- **10.** Partial differential equation (PDE): Introduction. Formation of P.D.E., Solution of PDE by Lagrange's method of solution and by Charpit's method. Jacobi's Method [5]

Question Pattern

Paper – VI (50 Marks)

Group A (15 marks) Analysis II

One question of 15 marks is to be answered out of 2 such question and each such question must have further part questions covering the entire syllabus.

Group B (35 marks) Differential Equations I

Five questions each carrying 7 marks are to be answered out of 9 questions covering the entire syllabus. Each such question may have further parts.

<u>SEMESTER – 4</u>

Paper VII

Group A (30 marks)

Real-Valued Functions of Several Real Variables

1. Point sets in two and three dimensions: Concept only of neighbourhood	d of a
point, interior point, limit point, open set, closed set.	[2]
2. Concept of functions on R ⁿ .	[1]
_	limits [3]
4. Functions R ² → R : Differentiability and its sufficient condition, differentiation map, chain rule, Euler's theorem and its converse. Commutativity of the sorder mixed partial derivatives : Theorems of Young and Schwarz.	
5. Jacobian of two and three variables, simple properties including fu dependence. Concept of Implicit function: Statement and simple applicat implicit function theorem for two variables Differentiation of Implicit function.	nction
6. Taylor's theorem for functions two variables. Lagrange's methor undetermined multipliers for function of two variables (problems only).	od of [3]
Group B (20 marks)	
Application of Calculus	
1. Tangents and normals : Sub-tangent and sub-normals. Angle of intersect curves. Pedal equation of a curve, pedal of a curve. [4]	ion of
2. Rectilinear asymptotes of a curve (Cartesian, parametric and polar for	rm [3]
3. Curvature-Radius of curvature, centre of curvature, chord of curvature, evo	
a curve.	[4]
4. Envelopes of families of straight lines and curves (Cartesian and para equations only).	metric. [4]
5. Concavity, convexity, singular points, nodes, cusps, points of inflexion,	
problems on species of cusps of a curve (Cartesian curves only).	51111p10 [5]
6. Familiarity with the figure of following curves: Periodic curves with so	
scaling, Cycloid, Catenary, Lemniscate of Bernoulli, Astroid, Cardiode, Foli	
Descartes, equiangular spiral.	[1]

7. Area enclosed by a curve, determination of C.G., moments and products of inertia

[3]

(Simple problems only).

Paper VII Question Pattern

Group - A (30 marks)

(Real-Valued Functions of Several Real Variables)

The pattern may be any one of the following:

(i) Three questions each of 10 marks are to be answered out of 5 given questions covering the entire syllabus. Each such question may contain further part questions.
(ii) Six questions each of 5 marks are to be answered out of 11 given questions covering the entire syllabus.

Group – B (20 marks)

(Application of Calculus)

Four questions each of 5 marks are to be answered out of 7 given questions covering the entire syllabus.

Paper VIII

Group A (25 marks)

Vector Calculus I

1. Vector differentiation with respect to a scalar variable, Vector functions of one scalar variable. Derivative of a vector. Second derivative of a vector. Derivatives of sums and products, Velocity and Acceleration as derivative. [5]

2. Concepts of scalar and vector fields. Direction derivative. Gradient, Divergence and curl, Laplacian and their physical significance. [5]

3. Line integrals as integrals of vectors, circulation, irrotational vector, work done, conservative force, potential orientation. Statements and verification of Green's theorem, Stokes' theorem and Divergence theorem. [8]

Group B (25 marks)

Analytical Dynamics of A Particle I

Recapitulations of Newton's laws. Applications of Newton's laws to elementary problems of simple harmonic motion, inverse square law and composition of two simple harmonic motions. Centre of mass. Basic kinematic quantities: momentum, angular momentum and kinetic energy. Principle of energy and momentum. Work and poser. Simple examples on their applications.

Impact of elastic bodies. Direct and oblique impact of elastic spheres. Losses of kinetic energy. Angle of deflection.

Tangent and normal accelerations. Circular motion. Radial and cross-radial accelerations.

Damped harmonic oscillator. Motion under gravity with resistance proportional to some integral power of velocity. Terminal velocity. Simple cases of a constrained motion of a particle.

Motion of a particle in a plane under different laws of resistance. Motion of a projectile in a resisting medium in which the resistance varies as the velocity. Trajectories in a resisting medium where resistance varies as some integral power of the velocity. [25]

Paper - VIII Question Pattern

Group - A (25 marks)

Vector Calculus I

Five questions each of 5 marks are to answered out of 9 given questions covering the entire syllabus.

Group - B (25 marks)

Analytical Dynamics of A Particle I

(i) One question of 7 marks are to answered out of 2 given questions.
(ii) Two questions each of 9 marks are to be answered out of 4 given questions covering the entire syllabus. Each such question may contain further part questions.

<u>SEMESTER – 5</u>

Paper IX

Real Analysis V (50 marks)

- **1.** Compactness in R : Open cover of a set. Compact set in R, a set is compact iff it is closed and bounded. [2]
- 2. Function of bounded variation (BV): Definition and examples. Monotone function is of BV. If f is on BV on [a,b] then f is bounded on [a,b]. Examples of functions of BV which are not continuous and continuous functions not of BV. Definition of variation function. Necessary and sufficient condition for a function f to be of BV on [a,b] is that f can be written as the difference of two monotonic increasing functions on [a,b]. Definition of rectifiable curve. A plane curve $\gamma = (f,g)$ is rectifiable if f and g both are of bounded variation (statement only). Length of a curve (simple problems only).

3. Riemann integration :

- (a) Partition and refinement of partition of a closed and bounded interval. Upper Darboux sum U (P,f) and lower Darboux sum L(P,f) and associated results. Upper integral and lower integral. Darboux's theorem. Darboux's definition of integration over a closed and bounded interval. Riemann's definition of integrability. Equivalence with Darboux definition of integrability (statement only). Necessary and sufficient condition for Riemann intergrability. [6]
- (b) Continuous functions are Riemann integrable. Definition of a set of measure zero (or negligible set or zero set) as a set covered by countable number of open intervals sum of whose lengths is arbitrary small. Examples of sets of measure zero: any subset of a set of measure zero, countable set, countable union of sets of measure zero. Concept of oscillation of a function at a point. A function is continuous at *x* if its oscillation at *x* is zero. A bounded function on a closed and bounded interval is Riemann integrable if the set of points of discontinuity is a set of measure zero (Lebesgue's theorem on Riemann integrable function). Problems on Riemann integrability of functions with sets of points of discontinuity having measure zero. [5]

Integrability of sum, scalar multiple, product, quotient, modulus of Riemann integrable functions. Properties of Riemann integrable functions arising from the above results. [3]

(d) Function defined by definite integral $\int_{a}^{x} f(t)dt$ and its properties.

Antiderivative (primitive or indefinite integral). [4]

(e) Fundamental theorem of integral calculus. First mean value theorem of integral calculus. Statement of second mean value theorems of integrals calculus (both Bonnet's and Weierstrass' form).

[2]

- **4.** Sequence and Series of functions of a real variable :
 - (a) Sequence of functions defined on a set: Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Dini's theorem on uniform convergence (statement only). Weierstrass's Mtest. [4]
 - (b) Limit function: Boundedness, Repeated limits, Continuity, Integrability and differentiability of the limit function of sequence of functions in case of uniform convergence. [5]
 - (c) Series of functions defined on a set: Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Dini's theorem on uniform convergence (statement only), Tests of uniform convergence Weierstrass' M-test. Statement of Abel's and Dirichlet's test and their applications. Passage to the limit term by term.
 - (d) Sum function: boundedness, continuity, integrability, differentiability of a series of functions in case of uniform convergence.
 - (e) Power series: Cauchy-Hadamard theorem. Determination of radius of convergence. Uniform and absolute convergence of power series. Properties of sum function. Abel's limit theorems. Uniqueness of power series having sum function. [8]

Paper X

Group A (20 marks)

Linear Algebra III

- Row space and column space of a matrix; null space and left null space; row rank, column rank and rank of a matrix. Fundamental theorem of linear algebra (Part I). Linear homogeneous system of equations:
 Solution space, related results using idea of rank, linear non-homogeneous system of equations----solvability and solution by Gauss-Jordan elimination. Free and basic variables, Pivots.
- 2. Inner product space: Definition and example, norm, Euclidian space, triangle inequality and Cauchy-Schwartz inequality, polarization identity, orthogonality of vectors, orthonormal basis, Gram-Schmidt process, orthogonal complement. Fundamental theorem of linear algebra (Part II). Every matrix transforms its row space into column space. [8]
- Eigen values and eigen vectors of matrices, Cayley-Hamilton theorem,
 Simple properties of eigen values and eigen vectors, real quadratic form (involving 3 variables). Reduction to normal form and classification (statement of relevant results only)
- 4. Linear transformation on vector spaces: Definition, Null space, range space, rank and nullity, rank-nullity theorem, simple applications, non-singular linear transformation, inverse of linear transformation. An m x n real matrix as a linear transformation from □ ⁿ to □ ^m. Transformation in □ ², matrices of rotation, projection, and reflection [4]
- 5. Natural isomorphism $\phi: V \longrightarrow \square^n$ (for an n-dimensional space V) and coordinate vector $[x]_{\alpha}$ with respect to ordered basis α of V. Matrices of linear transformations: $T: V_{\alpha} \longrightarrow W_{\beta}$ corresponds to unique $[T]_{\alpha}^{\beta}$ such that $[T(x)]_{\beta} = [T]_{\alpha}^{\beta} [x]_{\alpha}$ [finite dimensional cases]. Looking back to some simple matrix properties in the light of linear transformation: product of two matrices, rank of a matrix, inverse of a matrix. Change of basis, similarity of matrices. Diagonalization of a matrix (Statement and application of relevant results). Looking back at eigen values and eigen vectors. [8]

Group B (15 marks)

Tensor Calculus

A tensor as a generalized concept of a vector in an Euclidean space E^3 . To generalize the idea in an n-dimensional space. Definition of E^n . Transformation of co-ordinates in E^n (n = 2, 3 as example). Summation convention.

Contravariant and covariant vectors. Invariants. Contravariant, covariant and mixed tensors. The Kronecker delta. Algebra of tensors Symmetric and skew-symmetric tensors. Addition and scalar multiplication. Contraction. Outer and Inner products of tensors. Quotient law. Reciprocal Tensor. Riemannian space. Line element and metric tensor. Reciprocal metric tensor. Raising and lowering of indices with the help of metric tensor. Associated tensor. Magnitude of a vector. Inclination of two vectors. Orthogonal vectors. Christoffel symbols and their laws of transformations. Covariant differentiation of vectors and tensors.

[15]

Group C (15 marks)

Differential Equations II

- Laplace Transformation and its application in ordinary differential equations: Laplace Transform and Inverse Laplace Transform. Statement of Existence theorem. Elementary properties of Laplace Transform and its Inverse. Laplace Transform of derivatives. Laplace transform of integrals. Convolution theorem (Statement only). Application to the solution of ordinary differential equations of second order with constant coefficients. [4]
- **2.** Series solution at an ordinary point : Power Series solution of ordinary differential equations. Simple problems only. [2]

Paper XI

Group A (30 marks)

Analytical Statics

1.Friction: Law of Friction, Angle of friction, Cone of friction, To find the positions of equilibrium of a particle lying on a (i) rough plane curve, (ii) Rough surface under the action of any given forces. [4]

2.Astatic Equilibrium, Astatic Centre, Positions of equilibrium of a particle lying on a smooth plane curve under action of given force. Action at a joint in a framework. [4]

3.Centre of Gravity: General formula for the determination of C.G. Determination of position of C.G. of any arc, area of solid of known shape by method of integration.

[3]

- 4. Virtual work: Principle of virtual work for a single particle. Deduction of the conditions of equilibrium of a particle under coplanar forces from the principle of virtual work. The principle of virtual work for a rigid body. Forces which do not appear in the equation of virtual work. Forces which appear in the equation of virtual work. The principle of virtual work for any system of coplanar forces acting on a rigid body. Converse of the principle of virtual work.

 [8]
- 5. Stable and unstable equilibrium. Coordinates of a body and of a system of bodies. Field of forces. Conservative field. Potential energy of a system. The energy test of stability. Condition of stability of equilibrium of a perfectly rough heavy body lying on fixed body. Rocking stones. [6]
- 6. Forces in the three dimensions. Moment of a force about a line. Axis of a couple. Resultant of any two couples acting on a body. Resultant of any number of couples acting on a rigid body. Reduction of a system of forces acting on a rigid body. Resultant force is an invariant of the system but the resultant couple is not an invariant. Conditions of equilibrium of a system of forces acting on a body. Deductions of the conditions of equilibrium of a system of forces acting on a rigid body from the principle of virtual work. Poisnot's central axis. A given system of forces can have only one central axis. Wrench, Pitch, Intensity and Screw. Condition that a given system of forces may have a single resultant. Invariants of a given system of forces. Equation of the central axis of a given system of forces.

Group B (20 marks)

Analytical Dynamics of A Particle II

- 1. Central forces and central orbits. Typical features of central orbits. Stability of nearly circular orbits.
- 2. Planetary motion and Kepler's laws. Time of describing an arc of the orbit. Orbital energy. Relationship between period and semi-major axis. Motion of an artificial satellite.
- 3. Motion on a smooth curve under gravity. Motion of a rough curve under

- gravity e.g., circle, parabola, ellipse, cycloid etc.
- **4.** Varying mass problems. Examples of falling raindrops and projected rockets.
- 5. Linear dynamical systems, preliminary notions: solutions, phase portraits, fixed or critical points. Plane autonomous systems. Concept of Poincare phase plane. Simple examples of damped oscillator and a simple pendulum. The two-variable case of a linear plane autonomous system. Characteristic polynomial. Focal, nodal and saddle points. [20]

Paper XII

Group A (25 marks)

Hydrostatics

- **1.** Definition of Fluid. Perfect Fluid, Pressure. To prove that the pressure at a point in fluid in equilibrium is the same in every direction. Transmissibility of liquid pressure. Pressure of heavy fluids. To prove
 - (i) In a fluid at rest under gravity the pressure is the same at all points in the same horizontal plane.
 - (ii) In a homogeneous fluid at rest under gravity the difference between the pressures at two points is proportional to the difference of their depths.
 - (iii) In a fluid at rest under gravity horizontal planes re surfaces of equal density.
 - (iv) When two fluids of different densities at rest under gravity do not mix, their surface of separation is a horizontal plane.

Pressure in heavy homogeneous liquid. Thrust of heavy homogeneous liquid on plane surface.

- 2. Definition of centre of pressure. Formula for the depth of the centre of pressure of a plane area. Position of the centre of pressure. Centre of pressure of a triangular area whose angular points are at different depths. Centre of pressure of a circular area. Position of the centre of pressure referred to coordinate axes through the centroid of the area. Centre of pressure of an elliptical area when its major axis is vertical or along the line of greatest slope. Effect of additional depth on centre of pressure.
- **3.** Equilibrium of fluids in given fields of force: Definition of field of force, line of force. Pressure derivative in terms of force. Surface of equi-pressure. To find the necessary and sufficient conditions of equilibrium of a fluid under the action of a force whose components are *X*, *Y*, *Z* along the coordinate axes. To prove (i) that surfaces of equal pressure are the surfaces intersecting orthogonally the lines of force, (ii) when the force system is conservative, the surfaces of equal pressure are equi-potential surfaces and are also surfaces of equal density. To find the differential equations of the surfaces of equal pressure and density.
- **4.** Rotating fluids. To determine the pressure at any point and the surfaces of equal pressure when a mass of homogeneous liquid contained in a vessel, revolves uniformly about a vertical axis.
- **5.** The stability of the equilibrium of floating bodies. Definition, stability of equilibrium of a floating body, metacentre, plane of floatation, surface of buoyancy. General propositions about small rotational displacements. To derive the condition for stability.
- 6. Pressure of gases. The atmosphere. Relation between pressure, density and temperature. Pressure in an isothermal atmosphere. Atmosphere in convective equilibrium. [30]

Group B (25 marks)

Rigid Dynamics

- 1. Momental ellipsoid, Equimomental system. Principal axis. D'Alembert's principle. D'Alembert's equations of motion. Principles of moments. Principles of conservation of linear and angular momentum. Independence of the motion of centre of inertia and the motion relative to the centre of inertia. Principle of energy. Principle of conservation of energy.
- **2.** Equation of motion of a rigid body about a fixed axis. Expression for kinetic energy and moment of momentum of a rigid body moving about a fixed axis. Compound pendulum. Interchangeability of the points of a suspension and centre of oscillation. Minimum time of oscillation.
- **3.** Equations of motion of a rigid body moving in two dimensions. Expression for kinetic energy and angular momentum about the origin of rigid body moving in two dimensions. Necessary and sufficient condition for pure rolling. Two dimensional motion of a solid of revolution moving on a rough horizontal plane.
- 4. Equations of motion under impulsive forces. Equation of motion about a fixed axis under impulsive forces. To show that (i) of there is a definite straight line such that the sum of the moments of the external impulses acting on a system of particles about it vanishes, then the total angular momentum of the system about that line remains unaltered, (ii) the change of K.E. of a system of particles moving in any manner under the application of impulsive forces is equal to the work done by the impulsive forces. [30]

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Paper XIII

Group A (20 marks)

Real Analysis VI

- Improper Integral:
 - (a) Range of integration, finite or infinite. Necessary and sufficient condition for convergence of improper integral in both cases.
 - (b) Tests of convergence: Comparison and µ-test. Absolute and non-absolute convergence and interrelations. Abel's and Dirichlet's test for convergence of the integral of a product(statement only).
 - (c) Convergence and working knowledge of Beta and Gamma function and

their inter relation $\Gamma(n)\Gamma(n-1) = \frac{\pi}{\sin n\pi}$, 0<n<1 (to be assumed).

Computation of the integrals $\int_{0}^{\pi/2} \sin^{n} x dx$, $\int_{0}^{\pi/2} \cos^{n} x dx$, $\int_{0}^{\pi/2} \tan^{n} x dx$ when

they exist (using Beta and Gamma function).

[3]

- 2. Fourier series: Trigonometric series. Statement of sufficient condition for a trigonometric series to be a Fourier series. Fourier coefficients for periodic functions defined on $[-\pi, \pi]$. Statement of Dirichlet's conditions of convergence. Statement of theorem of sum of Fourier series.
- 3. Multiple integral: Concept of upper sum, lower sum, upper integral, lower integral and double integral (no rigorous treatment is needed). Statement of existence theorem for continuous functions. Change of order of integration. Triple integral. Transformation of double and triple integrals (Problem only). Determination of volume and surface area by multiple integrals (Problem only). [5]

Group B (15 marks) **Metric Space**

- 1. Definition and examples of metric spaces. Open ball. Open set. Closed set defined as complement of open set. Interior point and interior of a set. Limit point, derived set and closure of a set. Boundary point and boundary of a set. Properties of interior, closure and boundary of a set. Diameter of a set and bounded set. Distance between a point and a set. [7]
- 2. Subspace of a metric space. Convergent sequence. Cauchy sequence. Every Cauchy sequence is bounded. Every convergent sequence is Cauchy, not the converse. Completeness: definition and examples. Cantor intersection theorem. IR is a complete metric space. Q is not complete. [4]

Group C (15 marks)

Complex Analysis

- 1. Extended complex plane. Stereographic projection. [2]
- 2. Complex function: Limit, continuity and differentiability of complex functions. Cauchy-Riemann equations. Sufficient condition for differentiability of a complex function. Analytic functions. Harmonic functions. Conjugate harmonic functions. Relation between analytic function and harmonic function. [8]

Paper XIV

Group A (30 marks) Probability

Mathematical Theory of Probability:

Random experiments. Simple and compound events. Event space. Classical and frequency definitions of probability and their drawbacks. Axioms of Probability. Statistical regularity. Multiplication rule of probabilities. Bayes' theorem. Independent events. Independent random experiments. Independent trials. Bernouli trials and binomial law. Poisson trials. Random variables. Probability distribution, distribution function. Discrete and continuous distributions. Binomial, Poisson, Gamma, Uniform and Normal distribution. Poisson Process (only definition). Transformation of random variables. Two dimensional probability distributions. Discrete and continuous distributions in two dimensions. Uniform distribution and two dimensional normal distribution. Conditional distributions, transformation of random variables in two dimensions. Mathematical expectation. Mean, variance, moments, central moments. Measures of location, dispersion, skewness and kurtosis. Median, mode, function. Characteristic function. quartiles. Moment-generating Twodimensional expectation. Covariance, Correlation co-efficient. Joint characteristic function. Multiplication rule for expectations. Conditional expectation. Regression curves, least square regression lines and parabolas. Chisquare and t-distributions and their important properties (Statements only). Tchebycheff's inequality. Convergence in probability. Statements of : Bernoulli's limit theorem. Law of large numbers. Poissons's approximation to binomial distribution and normal approximation to binomial distribution. Concepts of asymptotically normal distribution. Statement of central limit theorem in the case of equal components and of limit theorem for characteristic functions (Stress should be more on the distribution function theory than on combinatorial problems. Difficult combinatorial problems should be avoided). [40]

Group B (20 marks) Statistics

Random sample. Concept of sampling and various types of sampling. Sample and population. Collection, tabulation and graphical representation. Grouping of data. Sample characteristic and their computation. Sampling distribution of a statistic. Estimates of a population characteristic or parameter. Unbiased and consistent estimates. Sample characteristics as estimates of the corresponding population characteristics. Sampling distributions of the sample mean and variance. Exact sampling distributions for the normal populations.

Bivariate samples. Scatter diagram. Sample correlation co-efficient. Least square regression lines and parabolas. Estimation of parameters. Method of maximum likelihood. Applications to binomial, Poisson and normal population. Confidence intervals. Interval estimation for parameters of normal population. Statistical hypothesis. Simple and composite hypothesis. Best critical region of a test. Neyman-Pearson theorem (Statement only) and its application to normal population. Likelihood ratio testing and its application to normal population. Simple applications of hypothesis testing.

Paper XV

Group A (25 marks)

Numerical Analysis

What is Numerical Analysis?

Errors in Numerical computation: Gross error, Round off error, Truncation error. Approximate numbers. Significant figures. Absolute, relative and percentage error.

Operators: , ∇ , E, μ , δ (Definitions and simple relations among them). Interpolation: Problems of interpolation, Weierstrass' approximation theorem only statement). Polynomial interpolation. Equispaced arguments. Difference table. Deduction of Newton's forward and backward interpolation formulae. Statements of Stirling's and Bessel's interpolation formulae. error terms. General interpolation formulae: Deduction of Lagrange's interpolation formula. Divided difference. Newton's General Interpolation formula (only statement). Inverse interpolation.

Interpolation formulae using the values of both f(x) and its derivative f(x): Idea of Hermite interpolation formula (only the basic concepts).

Numerical Differentiation based on Newton's forward & backward and Lagrange's formulae.

Numerical Integration: Integration of Newton's interpolation formula. Newton-Cote's formula. Basic Trapezoidal and Simpson's ½rd. formulae. Their composite forms. Weddle's rule (only statement). Statement of the error terms associated with these formulae. Degree of precision (only definition).

Numerical solution of non-linear equations: Location of a real root by tabular method. Bisection method. Secant/Regula-Falsi and Newton-Raphson methods, their geometrical significance. Fixed point iteration method.

Numerical solution of a system of linear equations: Gauss elimination method. Iterative method – Gauss- Seidal method. Matrix inversion by Gauss elimination method (only problems – up to 3×3 order).

Eigenvalue Problems: Power method for numerically extreme eigenvalues. Numerical solution or Ordinary Differential Equation: Basic ideas, nature of the problem. Picard, Euler and Runge-Kutta (4th order) methods (emphasis on the problems only). [30]

Group B (25 marks) Computer Programming

Fundamentals of Computer Science and Computer Programming:

Computer fundamentals: Historical evolution, computer generations, functional description, operating system, hardware & software.

Positional number system : binary, octal, decimal, hexadecimal system. Binary arithmetic.

Storing of data in a computer: BIT, BYTE, Word. Coding of data –

ASCII, EBCDIC, etc.

Algorithm and Glow Chart : Important features, Ideas about the complexities of algorithm. Application in simple problems.

Programming languages: General concepts, Machine language, Assembly Language, High Level Languages, Compiler and Interpreter. Object and Source Program. Ideas about some major HLL.

Introduction to ANSI C:

Character set in ANSI C. Key words: if, while, do, for, int, char, float etc. Data type: character, integer, floating point, etc. Variables, Operators: =, =, !!<, >, etc. (arithmetic, assignment, relational, logical, increment, etc.). Expressions: e.g. (a = = b)!! (b = = c), Statements: e.g. if (a>b) small = a; else small = b. Standard input/output. Use of while, if.... Else, for, do...while, switch, continue, etc. Arrays, strings. Function definition. Running simple C Programs. Header file. [30]

Boolean Algebra: Huntington Postulates for Boolean Algebra. Algebra of sets and Switching Algebra as examples of Boolean Algebra. Statement of principle of duality. Disjunctive normal and Conjunctive normal forms of Boolean Expressions. Design of simple switching circuits. [10]

Paper XVI

Practical [50 marks]

(Problem: 40, Sessional Work: 5, Viva: 5)

(A) Using Calculator

(1) **INTERPOLATION**:

Newton's forward & Backward Interpolation.

Stirling & Bessel's Interpolation.

Lagrange's Interpolation & Newton's Divided Difference Interpolation. Inverse Interpolation.

- (2) Numerical Differentiation based on Newton's Forward & Backward Interpolation Formulae.
- (3) Numerical Integration: Trapezoidal Rule, Simpson's 1/3 Rule and Weddle's Formula.
- (4) Solution of Equations: Bisection Method, Regula Falsi, Fixed Point Iteration. Newton-Raphson formula (including modified form for repeated roots and complex roots).
- (5) Solution of System of Linear Equations : Gauss' Elimination Method with partial pivoting, Gauss-Seidel/Jordon Iterative Method, Matrix Inversion.
- (6) Dominant Eigen pair of a (4×4) real symmetric matrix and least eigen value of a (3×3) real symmetric matrix by Power Method.
- (7) Numerical Solution of first order ordinary Differential Equation (given the initial condition) by :

Picard's Method, Euler Method, Heun's Method, Modified Euler's Method, 4th order Runge-Kutta Method.

(8) Problems of Curve Fitting: To fit curves of the form y=a+bx, y=a+bx+cx², exponential curve of the form y=ab^x, geometric curve y=ax^b by Least Square Method.

(B) ON COMPUTER:

The following problems should be done on computer using either C language:

- (i) To find a real root of an equation by Newton-Raphson Method.
- (ii) Dominant eigenpair by Power Method.
- (iii) Numerical Integration by Simpson's ½ Rule.
- (iv) To solve numerically Initial Value Problem by Euler's and RK₄ Method.

LIST OF BOOKS FOR REFERRENCE

Paper I Group A:

- 1. The Theory of Equations (Vol. I) – Burnside and Panton.
- Higher Algebra Barnard and Child. 2.
- 3. Higher Algebra (Classical) – S. K. Mapa

Paper I Group B; Paper IV Group A &B & Paper V Group A; Paper X Group A:

- First Course in Abstract Algebra Fraleigh.
- 2. 3. 4. 5. Topics in Algebra – Herstein.
- Topics in Abstract Algebra Sen Ghosh, Mukhopadhyay Higher Algebra (Abstract) S. K. Mapa Linear Algebra Hadley Test Book of Matrix B. S. Vaatsa

- 6.

Group A, Group B; Paper IV Group C: Paper II

- 1. Co-ordinate Geometry (Two dimensions)—S. L. Loney.
- 2. Co-ordinate Geometry of Three Dimensions – Robert J. T. Bell.
- 3. Advanced Analytical Geometry of Two & Three Dimensions – P. R. Ghosh, J. G. Chakravorty; and R. M. Khan
- 4. Elementary Treatise on Conic sections – C. Smith.
- 5. Solid Analytic Geometry – C. smith.
- 6. Higher Geometry – Efimov.

Paper III, Paper VI, Paper IX & Paper XIII Group A; Paper III Group B:

- 1. Basic Real & Abstract Analysis – Randolph J. P. (Academic Press).
- 2. A First Course in Real Analysis – M. H. Protter & G. B. Morrey (Springer Verlag, NBHM).
- 3. A Course of Analysis – Phillips.
- 4. Problems in Mathematical Analysis – B. P. Demidovich (Mir).
- 5. Problems in Mathematical Analysis – Berman (Mir).
- 6. Differential & Integral Calculus (Vol. I & II) – Courant & John.
- 7. Calculus of One Variable – Maron (CBS Publication).
- 8. Introduction to Real Analysis – Bartle & Sherbert (John Wiley & Sons.)
- 9. Mathematical Analysis – Parzynski.
- 10. Introduction to Real Variable Theory – Saxena & Shah (Prentice Hall Publication).
- 11. Real Analysis – Ravi Prakash & Siri Wasan (Tata McGraw Hill).
- 12. Mathematical Analysis – Shantinarayan (S. Chand & Co.).
- 13. Real Analysis – S. K. Mapa
- Theory & Applications of Infinite Series Dr. K. Knopp. 14.
- 15. Advanced Calculus – David Widder (Prentice Hall).
- 15 Charles Chapman Pugh: Real mathematical analysis; Springer; New York; 2002

- Sterling K. Berberian: A First Course in Real Analysis; Springer; New York; 1994
- 17 Steven G. Krantz: Real Analysis and Foundations; Chapman and Hall/CRC;

2004

- 18 Stephen Abbott: Understanding Analysis; Springer; New York, 2002
- T. M. Apostol: Mathematical Analysis, Addison-Wesley Publishing Co. 1957
- W. Ruddin: Principles of Mathematical Analysis, McGraw-Hill, New York, 1976
- J. F. Randolhp: Basic Real and Abstract Analysis, Academic Press; New York, 1968
- 22 Robert G Bartle, Donald R Sherbert: Introduction to real analysis; John Wiley Singapore; 1994
- 23 Integral Calculus Shanti Narayan & P. K. Mittal (S. Chand & Co. Ltd.)
- 24 Integral Calculus H. S. Dhami (New Age International)
- 25 Integral Calculus B. C. Das & B. N. Mukherjee (U. N. Dhur)
- 26 Differential & Integral Calculus (Vols. I & II) Courant & John.
- 27 Differential & Integral Calculus (Vol. I) N. Piskunov

Paper VII:

- 1. Differential Calculus Shantinarayan.
- 2. Integral Calculus Shantinarayan.
- 3. An elementary treatise on the Differential Calculus J. Edwards (Radha Publishing House).
- 4 Advanced Calculus David V. Widder (Prentice Hall)
- 5 Real Analysis Ravi Prakash & Siri Wasan (Tata McGraw Hill)
- 6 A Course of Analysis E. G. Phillips (Cambridge University Press)
- 7 Differential Calculus Shanti Naryaan (S. Chand & Co. Ltd.)
- 8 An elementary treatise on the Differential Calculus J. Edwards (Radha Publishing House)
- 9 Differential Calculus H. S. Dhami (New Age International)
- 10 Differential & Integral Calculus (Vols. I & II) Courant & John.
- 11 Differential & Integral Calculus (Vol. I) N. Piskunov (CBS Publishers & Distributors)

Paper II Group C, Paper VIII Group A:

- 1. Vector Analysis Louis Brand.
- 2. Vector Analysis Barry Spain.
- 3. Vector & Tensor Analysis Spiegel (Schaum).
- 4. Elementary Vector Analysis C. E. Weatherburn (Vol. I & II).

Paper V Group B:

- 1. Linear Programming: Method and Application S. I. Gass.
- 2. Linear Programming G. Hadley.
- 3. An Introduction to Linear Programming & Theory of Games S. Vajda.

- 4. Linear Programming & Game Theory J. G. Chakravorty & P. R. Ghosh
- 5. Linear Programming & Game Theory P. M. Karak
- 6. Linear Programming & Game Theory N. S. Kambo

Paper VI Group B & Paper X Group C:

- 1. Differential Equations Lester R. Ford (McGraw Hill).
- 2. Differential Equations S. L. Ross (John Wiley).
- 3. Differential Equations H. T. H. Piaggio.
- 4. A Text Book of Ordinary Differential Equations Kiseleyev, Makarenko & Krasnov (Mir).
- 5. Differential Equations H. B. Phillips (John Wiley & Sons).
- 6. Differential Equations with Application & Programs S. Balachanda Rao, H. R. Anuradha (University Press).
- 7. Text Book of Ordinary Differential Equations (2nd Ed.) S. G. Deo, V. Lakshmikantham & V. Raghavendra (Tata McGraw Hill).
- 8. An Elementary Course in Partial Differential Equation T. Amarnath (Narosa).
- 9. An Introductory Course on Ordinary Differential Equation D. A. Murray.
- 10. Differential Equations R. K. Ghosh & K. Maity
- 11. Differential Equations J. G. Chakravorty & P. Ř. Ghosh

Paper VIII Group B & Paper XI Group B; Paper XII Group B:

- An Elementary Treatise on the Dynamics of a Particle & of Rigid bodies
 S. L. Loney (Macmillan).
- 2. Analytical Dynamics of a particle J. G. Chakravorty & P. R. Ghosh
- 3. Analytical Dynamics of a particle Dutta & Jana
- 4. Analytical Dynamics of a particle Ganguly and Saha
- 5. Dynamics of Rigid Bodies Nand, Tyagi & Sharma
- 6. Dynamics of Rigid Bodies S. A. Mollah

Paper X Group B, Group C:

- 1. Tensor Calculus (Schaum Series) Spiegel.
- 2. Tensor Calculus Barry Spain
- 3. Tensor Calculus & Riemannian Geometry J. K. Goyel & K. P. Gupta
- 4. Tensor Calculus M. C. Chaki
- 5. Differential Equations & Laplace Transforms S. Sankarappan & S. Kalavathy
- 6. A Course of Advanced Calculus A. K. Sarkar & N. Mandal
- 7. An use of integral transforms I. N. Sneddon
- 8. Elementary Treatise on Laplace Transform B. Sen (World Press).

Paper XI Group A:

1. Analytical Statics – S. L. Loney

- 2. Analytical Statics M. C. Ghosh
- 3. Analytical Statics S. Pradhan & S. Sinha
- 4. Analytical Statics S. A. Mollah

Paper XII Group A, Group B:

- 1. Hydrostatics J. M. Kar
- 2. Hydrostatics A. S. Ramsey

Paper XIII Group B, Group C:

- 1. L. V. Ahlfors: Complex Analysis: an introduction to the theory of analytic functions of one complex variable; McGraw-Hill; New York;1966
- 2. S. Ponnusamy: Foundations of Complex Analysis; Narosa; New Delhi; 1995
- 3. R. V. Churchill and J.W.Brown: Complex Variables and Applications; Mcgraw-Hill; New York; 1996
- 4. P. K. Jain and K. Ahmad: Metric Spaces, Narosa Publishing House; New Delhi; 1996

Paper XIV:

- 1. The elements of probability theory and some of its applications H. Cramer.
- 2. An introduction to probability theory and its applications (Vol. 1) W. Feller.
- 3. Mathematical methods of statistics H. Cramer.
- 4 Theory of probability B. V. Gnedenko.
- 5. Mathematical probability J. V. Uspensky.

Paper XV Group A, Group B:

- 1. Numerical methods E. Balagurusamy (Tata McGraw Hill).
- 2. Let us C Y. Kanetkar (BPB Publications).
- 3. Programming in C V. Krishnamoorthy and K. R. Radhakrishnan (Tata Mcgraw Hill).
- 4. C by example : Noel Kalicharan (Cambridge University Press).
- 5. Programming in ANSI C E. Balagurusamy (Tata McGraw Hill).
- 6. Introduction to numerical analysis F. B. Hilderbrand (TMH Edition).
- 7. Numerical Analysis J. Scarborough.
- 8. Introduction to numerical analysis Carl Erik Froberg (Addison Wesley Publishing).
- 9. Numerical methods for science and engineering R. G. Stanton (Prentice Hall).

Paper XVI:

1. C Language and Numerical Methods – C. Xaviers

B. Sc. MATHEMATICS GENERAL (MTMG) SYLLABUS

(SEMESTER SYSTEM)

(w. e. f. July, 2012)

SEMESTER - I

PAPER – I: Group A: Classical Algebra (20 marks)

Group B: Analytical Geometry of Two Dimensions (15 marks)

Group C: Vector Algebra (15 marks)
Group D: Differential Calculus (25 marks)

SEMESTER - II

PAPER – II : Group A : Integral Calculus (10 marks)

Group B : Differential Equations (15 marks)

Group C: Modern Algebra (25 marks)

Group D: Analytical Geometry of Three Dimensions

(25 marks)

SEMESTER - III

PAPER – III: Group A: Differential Calculus (25 marks)

Group B: Numerical Methods (20 marks) **Group C:** Linear Programming (30 marks)

SEMESTER - IV

PAPER – IV: Group A: Integral Calculus (15 marks)

Group B: Differential Equations (10 marks)Group C: Analytical Dynamics (50 marks)

SEMESTER - I

PAPER – I (75 marks)

Group - A (20 marks) Classical Algebra

- 1. Complex Numbers: De Moivre's Theorem and its applications. Exponential, Sine, Cosine and Logarithm of a complex number. Definition of a ^z, (a≠0). Inverse circular and Hyperbolic functions.
- **2. Polynomials:** Fundamental Theorem of Classical Algebra (Statement only). Polynomials with real co-efficient: The *n*th degree polynomial equation has exactly *n* roots. Nature of roots of an equation (Surd or Complex roots occur in pairs). Statement of Descartes' Rule of signs and its applications. Statements of:
 - (i) If the polynomial f(x) has opposite signs for two real values of x, e.g. a and b, the equation f(x) = 0 has an odd number of real roots between a and b; if f(a) and f(b) are of same sign, either no real root or an even number of roots lies between a and b.
 - (ii) Rolle's Theorem and its direct applications.

Relation between roots and co-efficients. Symmetric functions of roots, Transformations of equations. Cardin's method of solution of a cubic.

- **3. Determinant up to the third order:** Properties, Cofactor and Minor. Product of two determinants. Adjoint, Symmetric and Skew-symmetric determinants. Solutions of linear equations with not more than three variables by Cramer's Rule.
- **4. Matrices of Real Numbers:** Equality of matrices. Addition of matrices. Multiplication of a matrix by a scalar. Multiplication of matrices Associative properties. Transpose of matrix its properties. Inverse of a non-singular square matrix. Symmetric and Skew-symmetric matrices. Scalar matrix. Orthogonal matrix. Elementary operations on matrices.

Rank of a matrix: Determination of rank either by considering minors or by sweep-out process. Consistency and solution of a system of linear of equations with not more than 3 variables by matrix method.

Group B (15 marks) Analytical Geometry of Two Dimensions

- **1. Transformations of Rectangular axes:** Translation, Rotation and their combinations. Invariants.
- **2. General equation of second degree in** *x* **and** *y***:** Reduction to canonical forms. Classification of conic.
- 3. **Pair of straight lines:** Condition that the general equation of second degree in x and y may represent two straight lines. Points of intersection of two intersecting straight lines. Angle between two lines given by $ax^2 + 2hxy + by^2 = 0$. Equation of bisectors. Equation of two lines joining the origin to the points in which a line meets a conic.
- 4. Equations of pair of tangents from an external point, chord of contact,

- **poles and polars in case of General conic:** Particular cases for Parabola, Ellipse, Circle, Hyperbola.
- **5. Polar equation** of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equation of chord joining two points. Equations of tangent and normal.

Group C (15 marks) Vector Algebra

Addition of Vectors. Multiplication of a Vector by a scalar. Collinear and Coplanar Vectors. Scalar and Vector products of two and three vectors. Simple applications to problems of Geometry. Vector equation of plane and straight line. Volume of Tetrahedron. Application to problems of Mechanics (Work done and Moment).

Group D (25 marks) Differential Calculus

- **1.** Rational Numbers. Geometrical representation. Irrational number. Real number represented as point on a line Linear Continuum. Acquaintance with basic properties of real number (No deduction or proof is included).
- **2.** Real-valued functions defined on an interval: Limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval. Acquaintance (no proof) with the important properties of continuous functions on closed intervals. Statement of existence of inverse function of a strictly monotone function and its continuity.
- Derivative its geometrical and physical interpretation. Sign of derivative

 Monotonic increasing and decreasing functions. Relation between continuity and derivability. Differential application in finding approximation.
- **4.** Successive derivative Leibnitz's Theorem and its application.
- **5. Application of the principle of Maxima and Minima** for a function of single variable in geometrical, physical and other problems.
- **6. Functions of two and three variables:** Their geometrical representations. Limit and Continuity (definitions only) for functions of two variables. Partial derivatives: Knowledge and use of Chain Rule. Exact differentials (emphasis on solving problems only). Functions of two variables Successive partial derivatives: Statement of Schwarz's Theorem on commutative property of mixed derivatives. Euler's theorem on homogeneous function of two and three variables. Maxima and minima of functions of not more than three variables Lagrange's Method of undetermined multiplier Problems only. Implicit function in case of function of two variables (existence assumed) and derivative.
- **7. Applications of Differential Calculus:** Tangents and Normals, Pedal equation and Pedal of a curve. Rectilinear Asymptotes (Cartesian only). Definition and examples of singular points (viz. Node, Cusp, Isolated point).

Question Pattern (MTMG, Paper – I)

Group - A (20 marks)

Five questions, each carrying 4 marks are to be answered out of eight questions.

There will be two questions from each topic.

Group - B (15 marks)

Three questions, each carrying 5 marks are to be answered out of five questions covering all the topics.

Group – C (15 marks)

Three questions, each carrying 5 marks are to be answered out of five questions covering all the topics.

Group - D (25 marks)

There will be one question of 5 marks with two parts, one carrying 2 marks and one carrying 3 marks. Each part will have at least one alternative.

Two questions, each of 10 marks, are to be answered out of 4 questions covering all the topics. Each question will have further parts.

SEMESTER - II

PAPER – II (75 marks)

Group A (10 marks) <u>Integral Calculus</u>

- 1. Integrations of the form:
 - $\int \frac{dx}{a+b\cos x}$, $\int \frac{l\sin x + m\cos x}{n\sin x + p\cos x} dx$ and Integration of Rational functions.
- 2. Evaluation of definite integrals.
- **3.** Integration as the limit of a sum (with equally spaced as well as unequal intervals)

Group B (15 marks) Differential Equations

- **1.** Order, degree and solution of an ordinary differential equation (ODE) in presence of arbitrary constants. Formation of ODE. First order equations:
 - (i) Variables separable.
 - (ii) Homogeneous equations and equations reducible to homogeneous forms.
 - (iii) Exact equations and those reducible to such equation.
 - (iv) Euler's and Bernoulli's equations (Linear).
 - (v) Clairaut's Equations: General and Singular solutions.
 - 2. Simple applications: Orthogonal Trajectories.

Group C(25 marks) Mode<u>rn Algebra</u>

- **1. Basic concept :** Sets, Sub-sets, Equality of sets, Operations on sets : Union, intersection and complement. Verification of the laws of Algebra of sets and De Morgan's Laws. Cartesian product of two sets. Mappings, One-One and onto mappings. Composition of Mappings—concept only, Identity and Inverse mappings. Binary Operations in a set. Identity element. Inverse element.
- **2. Introduction of Group Theory :** Definition and examples taken from various branches (examples from number system, roots of unity, 2 x 2 real matrices, non-singular real matrices of a fixed order). Elementary properties using definition of Group. Definition and examples of sub-group Statement of necessary and sufficient condition its applications.
- **3. Definitions and examples** of (i) Ring, (ii) Field, (iii) Sub-ring, (iv) Subfield.

- **4.** Concept of Vector space over a Field: Examples, Concepts of Linear combinations, Linear dependence and independence of a finite set of vectors, Sup-space. Concepts of generators and basis of a finite-dimensional vector space. Problems on formation of basis of a vector space (No proof required).
- **5.** Real Quadratic Form involving not more than three variables Problems only.
- **6.** Characteristic equation of a square matrix of order not more than three determination of Eigen Values and Eigen Vectors Problems only. Statement and illustration of Cayley-Hamilton Theorem.

Group D (25 marks) Analytical Geometry of three dimensions

- 1. Rectangular Cartesian co-ordinates: Distance between two points. Division of a line segment in a given ratio. Direction cosines and direction ratios of a straight line. Projection of a line segment on another line. Angle between two straight lines.
- **2. Equation of a Plane :** General form. Intercept and Normal form. Angle between two planes. Signed distance of a point from a plane. Bisectors of angles between two intersecting planes.
- **3. Equations of Straight line :** General and symmetric form. Distance of a point from a line. Coplanarity of two straight lines. Shortest distance between two skew-lines.
- **4. Sphere** and its tangent plane.
- 5. Right circular cone.

Question Pattern (MTMG, Paper – II)

Group – A (10 marks)

One question of 2 marks is to be answered out of two questions.

Two questions each of 4 marks are to be answered out of four questions. Each such question may have further parts

$$Group - B$$
 (15 marks)

Two questions, each carrying 2 marks are to be answered out of three questions.

One question of 3 marks is to be answered out of two questions.

Two questions each of 4 marks are to be answered out of four questions. Each such question may have further parts.

Three questions, each carrying 3 marks are to be answered out of five questions.

Four questions each of 4 marks are to be answered out of 6 questions. Each such question may have further parts.

Three questions, each carrying 3 marks are to be answered out of five questions.

Four questions each of 4 marks are to be answered out of 6 questions. Each such question may have further parts.

SEMESTER – III

PAPER – III (75 marks)

Group A (25 marks) Differential Calculus

1. Statement of Rolle's Theorem and its geometrical interpretation. Mean Value Theorems of Lagrange and Cauchy. Statements of Taylor's and Maclaurin's Theorems with Lagrange's and Cauchy's form of remainders. Taylor's and Maclaurin's Infinite series for functions like e^x, sin x, cos x. (1+x)ⁿ, log(1+x) [with restrictions wherever necessary]

Indeterminate Forms: L'Hospital's Rule: Statement and problems only.

- **2. Sequence :** Definition of bounds of a sequence and monotone sequence. Limit of a sequence. Statements of limit theorems. Concept of convergence and divergence of monotone sequences applications of the theorems, in particular, definition of *e*. Statement of Cauchy's general principle of convergence and its application.
- **3. Infinite series of constant terms:** Convergence and Divergence (definitions). Cauchy's principle as applied to infinite series (application only). Series of positive terms: Statements of Comparison test, D'Alembert's Ratio test. Cauchy's nth root test and Raabe's test Applications. Alternating series: Statement of Leibnitz test and its applications.

Group B (20 marks) **Numerical Methods**

- **1.** Approximate numbers, Significant figures, Rounding off numbers. Error Absolute, Relative and Percentage.
 - **2. Operators** , ∇ and E (Definitions and some relations among them).
 - **3. Interpolation :** The problem of Interpolation, Equispaced arguments Difference Tables, Deduction of Newton's Forward Interpolation Formula. Remainder term (expression only). Newton's Backward Interpolation formula (statement only) with remainder term. Unequally spaced arguments Lagrange's Interpolation Formula (statement only). Numerical problems on Interpolation with both equi- and unequally-spaced arguments.
- **4. Number Integration :** Trapezoidal and Simpson's ¹/₃rd formula (statement only). Problems on Numerical Integration.
- **5. Solution of Numerical Equation :** To find a real root of an algebraic or transcendental equation. Location of root (Tabular method), Bisection method. Newton-Raphson method with geometrical significance. Numerical problems.

(Note: emphasis should be given on problems)

Group C (30 marks) Linear Programming

1. Motivation of Linear Programming problem. Statement of L.P.P. formulation of L.P.P. Slack and Surplus variables. L.P.P. is matrix form. Convex set, Hyperplane, Extreme points, Convex Polyhedron, Basic solutions and Basic Feasible Solutions (B.F.S.) Degenerate and Non-degenerate B.F.S.

The set of all feasible solutions of an L.P.P. is a convex set. The objective function of an L.P.P. assumes its optimal value at an extreme point of the convex set of feasible solutions. A B.F.S. to an L.P.P. corresponds to an extreme point of the convex set of feasible solutions.

Fundamental Theorem of L.P.P. (Statement only). Reduction of a feasible solution to a B.F.S. Standard form of an L.P.P. Solution by graphical method (for two variables), by simplex method and method of penalty. Concept of duality. Duality theory. The dual of the dual is the primal. Relation between the objective values of dual and the primal problems. Dual problems with at most one unrestricted variable, one constraint of equality.

Transportation and Assignment problem and their optimal solutions.

<u>Question Pattern (MTMG, Paper – III)</u>

Group - A (25 marks)

One compulsory question of 5 marks and two other questions of 10 marks each are to be answered following the norms mentioned below:

- (1) The compulsory question will contain conceptual / objective type questions (viz. "correct or justify", "prove or disprove", "true or false", etc.) of marks 2 and 3 respectively, having alternatives / optional in both the cases.
- (2) Two questions each of 10 marks are to be answered out of four questions. Each such question will contain further parts.

Group - B (20 marks)

Two questions, each carrying 2 marks are to be answered out of four questions.

Two questions each of 8 marks are to be answered out of four questions. Each such question may have further parts.

Group – C (30 marks)

Three questions, each carrying 2 marks are to be answered out of five questions.

Two questions each of 12 marks are to be answered out of four questions. Each such question may have further parts.

SEMESTER – IV

PAPER - IV (75 marks)

Group A (15 marks)

Integral Calculus

- 1. Reduction formulae of $\int \sin x \cos x \, dx$, $\int \frac{\sin x}{\cos^n x} \, dx$, $\int \tan x \, dx$ and associated problems (*m* and *n* are non-negative integers).
- **2. Definition of Improper Integrals :** Statements of (i) μ -test, (ii) Comparison test (Limit form excluded) Simple problems only. Use of Beta and Gamma functions (convergence and important relations being assumed).
- **3.** Working knowledge of Double integral.
- **4. Applications :** Rectification, Quadrature, Volume and Surface areas of solids formed by revolution of plane curve and areas Problems only.

Group B (10 marks) Differential Equations

Second order linear equations : Second order linear differential equations with constant. Coefficients. Euler's Homogeneous equations.

Group C (50 marks) **Analytical Dynamics**

- 1. Velocity and Acceleration of a particle. Expressions for velocity and acceleration in rectangular Cartesian and polar co-ordinates for a particle moving in a plane. Tangential and normal components of velocity and acceleration of a particle moving along a plane curve.
- **2. Concept of Force :** Statement and explanation of Newton's laws of motion. Work, power and energy. Principles of conservation of energy and momentum. Motion under impulsive forces. Equations of motion of a particle (i) moving in a straight line, (ii) moving in a plane.
- **3.** Study of motion of a particle in a straight line under (i) constant forces, (ii) variable forces (S.H.M., Inverse square law, Damped oscillation, Forced and Damped oscillation, Motion in an elastic string). Equation of Energy. Conservative forces.
- **4. Motion in two dimensions :** Projectiles in vacuo and in a medium with resistance varying linearly as velocity. Motion under forces varying as distance from a fixed point.
- **5.** Central orbit. Kepler's laws of motion. Motion under inverse square law.

Question Pattern (MTMG, Paper – IV)

Group - A (15 marks)

One question of 3 marks is to be answered out of three questions.

Two questions each of 6 marks are to be answered out of four questions covering the entire syllabus. Each such question may have further parts

Group – B (10 marks)

One question of 2 marks is to be answered out of two questions.

Two questions each of 4 marks are to be answered out of four questions covering the entire syllabus. Each such question may have further parts.

Group - C (50 marks)

Four questions, each carrying 2 marks are to be answered out of six questions.

Six questions each of 7 marks are to be answered out of eleven questions covering the entire syllabus. Each such question may have further parts.