Ramakrishna Mission Residential College

(Autonomous)

Narendrapur, Kolkata – 700103

Department of Mathematics

Syllabus for three-year B.Sc.

In

Mathematics Honours

Under **CBCS**

2018

(Revised in 2021)

Name of the Programme	Programme Code
B. Sc. Mathematics Honours	BSHMAT

Program Objective

Honours – Mathematics, though not a science per se, is often referred to as the mother of all sciences. Its importance for a student willing to pursue basic science can never be underestimated. The principal objective of our syllabus is to empower the young learners, who aspire to make a career in teaching and/or research in Mathematics or any other related science subjects in future, so that their transition to the next higher step in mathematics learning may be as smooth as possible, through helping them to master a solid foundational knowledge of the basic theories from various branches of modern mathematics.

Generic Elective – This program is designed for the students who take Mathematics as an elective subject along with their chosen honours subject. Some of these honours subjects like Physics / Chemistry / Statistics / Computer Science / Economics specifically require some mathematical knowledge back-up, which is far from the rudimentary knowledge of mathematics that is provided at the plus two levels in general. Hence this course is designed to cater to this specific need by chalking out a common minimum requirement of the above mentioned disciplines, as far as practicable.

Course Structure: Semester-wise distribution of Courses

Honours

Semester	Course Name	Course Code	Credits
1	Classical Algebra , Abstract Algebra – I	HMAT1CC01N	6
	Analytical Two Dimensional Geometry , Differential Equations – I	HMAT1CC02N	6
2	Real Analysis – I	HMAT2CC03N	6
	Abstract Algebra – II, Linear Algebra – I	HMAT2CC04N	6
3	Analytical Three Dimensional Geometry with Vector Algebra, Multivariate Calculus – I	HMAT3CC05N	6
	Linear Algebra – II, Application of Calculus	HMAT3CC06N	6
	Real Analysis – II	HMAT3CC07N	6
4	Analytical Mechanics: Analytical Statics, Analytical Dynamics	HMAT4CC08N	6
	Linear Algebra – III	HMAT4CC09N	6
	Multivariate Calculus – II (Including vector calculus), Complex Analysis	HMAT4CC10N	6
5	Real Analysis – III, Differential Equations – II (including PDE)	HMAT5CC11N	6
	General Topology	HMAT5CC12N	6
	See DSE	HMAT5DS11L HMAT5DS12N	6
	See DSE	HMAT5DS21N HMAT5DS22N	6
6	Probability and Statistics	HMAT6CC13N	6
	Numerical Methods, Numerical Methods Lab	HMAT6CC14L	6
	See DSE	HMAT6DS31N HMAT6DS32N	6
	See DSE	HMAT6DS41N HMAT6DS42N	6
	Grand Total		108

Discipline Specific Electives (DSE)*

DSE 1 (Semester 5)	DSE 2 (Semester 5)	DSE 3 (Semester 6)	DSE 4 (Semester 6)
1. Computer Programming with C & Scientific Computing with R [HMAT5DS11L] 2. Graph Theory & Number Theory [HMAT5DS12N]	1. Advanced Algebra [HMAT5DS21N] 2. Advanced Mechanics [HMAT5DS22N]	1. L.P.P. & Game Theory [HMAT6DS31N] 2. Advanced Analysis [HMAT6DS32N]	1. Mathematical Logic [HMAT6DS41N] 2. Differential Geometry [HMAT6DS42N]

*A student has to opt for any one of the courses available/given by the department in a specific year under each category.

Semester-wise detailed syllabus

SEMESTER – 1		
Name of the Paper : **CC01**		
Name of the Course : Classical Algebra, Abstract Algebra – I		
Course Code : HMAT1CC01N		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course a student will acquire knowledge on

- complex numbers, theory of equations and certain important inequalities and will be able to solve problems related to those topics.
- rudimentary theory of numbers.
- basic group theory and skill to solve the related problems that will make a foundation for the second part of this course to be given in Semester – 2.

Detailed Syllabus

Group – A [40 Marks]
(Classical Algebra)

1. **Integers**: Well–ordering property of positive integers, Second Principle of Mathematical Induction. Division Algorithm ($a = bq + r$, $b \neq 0$, $0 \leq r < |b|$). The greatest common divisor (a, b) of two integers a and b. Existence and uniqueness of (a, b). Relatively prime integers. The equation $ax + by = c$ has integral solution if and only if (a, b) divides c (a, b, c are integers). Congruence relation modulo n. Prime integers. Euclid's first theorem: If some prime p divides ab, then p divides either a or b. Euclid's second theorem – There are infinitely many prime integers. Unique factorization theorem. Linear Congruences. Fundamental Theorem of Arithmetic. Chinese Remainder Theorem and simple problems. Little Theorem of Fermat & Euler's generalization. Arithmetic functions, some arithmetic functions such as φ, τ, σ and their properties.
2. **Complex Numbers**: De-Moivre's Theorem and its applications, Exponential, Sine, Cosine and Logarithm of a complex number. Definition of a^z ($a \neq 0$). Inverse circular and hyperbolic functions.
3. **Theory of Equations**: Polynomials with real co-efficients. Fundamental Theorem of Classical Algebra (statement only). The n-th degree polynomial equation has exactly n roots. Nature of roots of an equation (surd or complex roots occur in pairs). Statements of Descartes' rule of signs. Sturm's Theorem and their applications. Multiple roots. Relation between roots and coefficients. Symmetric functions of roots. Transformation of equations.

Reciprocal equations. Cardan's method of solving a cubic equation. Ferrari's method of solving a biquadratic equation. Binomial equation. Special roots.

4. **Inequality**: Inequalities $AM \geq GM \geq HM$ and their generalizations. The theorem of weighted means and m-th power theorem. Cauchy's inequality (statement only) and its direct applications.

Group – B [25 Marks]
(Abstract Algebra – I)

1. **Relation & Function**: Revision of basic ideas on relations and functions. Fundamental theorem of equivalence relation, residue class of integers $\mathbb{Z}_n$. Partial order relation, poset, linear order relation. Invertibility and inverse of a mapping.
2. **Groups**: Semigroup, Group, Abelian group. Various examples of groups (Klein's 4-group, group of residue classes of integers, group of symmetries, dihedral group, quaternion group [through matrices]), elementary properties, conditions for a semigroup to be a group, integral power of elements and order of an elements in a group, order of a group. Permutation groups, cycle, transposition, every permutation can be expressed as a product of disjoint cycles (statement only), even and odd permutations, symmetric group, alternating group.
3. **Subgroups:** Necessary and sufficient condition for a subset of a group to be a subgroup. Intersection and union of subgroups. Necessary and sufficient condition for union of two subgroups to be a subgroup. Normalizer, Centralizer, center of a group. Product of two subgroups. Generator of a group, examples of finitely generated groups, cyclic groups, Subgroups of cyclic groups. Necessary and sufficient condition for a finite group to be cyclic. Cosets and Lagrange's theorem on finite group; counter example to establish the failure of converse. Fermat's little theorem, Normal subgroup of a group and its properties. Quotient group.

References:

1. Titu Andrescu and Dorin Andrica – Complex Numbers from A to Z, Springer (2014)
2. W S Burnside and A W Panton – Theory of Equations (Vol. 1), University Press, Dublin (1924)
3. S K Mapa – Higher Algebra (Classical), Sarat Book Distributors, Kolkata (8th Edition, 2011)
4. Gareth Jones and Josephine M Jones – Elementary Number Theory, Springer (1998)
5. Titu Andrescu and Dorin Andrica – Number Theory, Springer (2009)
6. M K Sen, S Ghosh, P Mukhopadhyay and S Maity – Topics in Abstract Algebra, University Press, Hyderabad (3rd Edition, 2018)
7. D S Malik, John M Mordeson and M K Sen – Fundamentals of Abstract Algebra, McGraw Hill, New York (1997)
8. John B Fraleigh – A First Course in Abstract Algebra, Pearson (7th Edition, 2002)
9. M Artin – Abstract Algebra, Pearson (2nd Edition, 2011)
10. Joseph A Gallian – Contemporary Abstract Algebra, Narosa Publishing House, New Delhi (4th Edition, 1999)
11. Joseph J Rotman – An Introduction to the Theory of Groups, Springer (4th Edition, 1995)
12. I N Herstein – Topics in Algebra, Wiley Eastern Limited, India (1975)

Question Pattern for Semester – 1 Examinations

(Course Code: HMAT1CC01N & Paper: CC01)

Group – A (Classical Algebra, 40 marks)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

(ii) Answer **two** questions of 5 marks each from 3 given questions from Article – 1.

(iii) Answer **one** question of 6 marks from 2 given questions from Article – 2.

(iv) Answer **two** questions of 5 marks each from 3 given questions from Article – 3

(v) Answer **one** questions of 4 marks from 2 given questions from Article – 4.

Group – B (Abstract Algebra – I, 25 marks)

(i) Answer **two** objective / MCQ type questions of 2 marks each from 3 given questions.

(ii) Answer **five** questions of 3 marks each from 7 given questions.

(iii) Answer **one** question of 6 marks from 2 given questions. Each question may contain further parts.

SEMESTER – 1		
Name of the Paper : **CC02**		
Name of the Course : Analytical Two Dimensional Geometry, Differential Equations – I		
Course Code : HMAT1CC02N		
Full Marks : 100		Credit : 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course a student will acquire knowledge

- on two dimensional analytical geometry (both Cartesian and Polar coordinate system) and will be able to solve problems related to those topics, as a continuation of their previous concepts from plus two level.
- on different techniques towards solving first order ordinary differential equations and their applications, as a continuation of their previous concepts from plus two level.
- and skill towards solving various second order ordinary differential equations, simultaneous linear differential equations, eigenvalue problems, which are of paramount importance as a tool, not only in mathematics but also in almost all allied science subjects.

Detailed Syllabus

Group – A [30 Marks]

(Analytical Two Dimensional Geometry)

1. **Transformation of Rectangular axes**: Translation, Rotation and their combination (rigid motion). Theory of Invariants.
2. **Pair of straight lines**: Condition that the general equation of second degree in two variables may represent a pair of straight lines. Point of intersection of a pair of intersecting straight lines. Angle between a pair of straight lines given by $ax^2 + 2hxy + by^2 = 0$. Angle bisectors. Properties of the pair of straight lines of the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. Equation of a pair of straight lines joining the origin to the points in which a straight line meets a conic.
3. **General equation of second degree in two variables**: Reduction into canonical form. Classification of conics, Lengths and position of the axes.
4. **Circle, Parabola, Ellipse and Hyperbola**: Equations of pair of tangents from an external point, chord of contact, poles and polars, conjugate points and conjugate lines.
5. **Polar equation**: Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equations of tangent, normal, chord of contact.

Group – B (35 Marks)
[Differential Equations – I]

1. **First order ordinary differential equations:** The existence and uniqueness theorem of Picard (statement only). Exact differential equations. Condition of exactness. Integrating factor. Rules for finding integrating factor (statement of relevant results only). Special integrating factors and transformations.

2. **Linear differential equations of first order**: Linear differential equations, Bernoulli's equation, Equations reducible to linear form.

3. **First order but higher degree ordinary differential equations**: Equations solvable for x, y and p, where $p = \frac{dy}{dx}$. Clairaut's differential equations - Their general and singular solutions.

4. **Linear differential equations of higher order:** Wronskian, its properties and applications. Complementary function and particular integral. Symbolic operator D. Solution by operator method using operator D, method of variation of parameters. Cauchy-Euler homogeneous differential equation and its reduction to an equation of constant coefficients. Equations reducible to Cauchy-Euler equation.

5. **Initial value and boundary value problems of second order:** Simple Eigen Value Problems.

6. **Simultaneous linear differential equations**: Solution by method of elimination, method of differentiation and method of determinant.

7. **Power series solution**: Solution of an ordinary differential equation about an ordinary point and a regular singular point (up to second order).

References:

1. S L Loney – Co-ordinate Geometry
2. N Mandal and B Pal – Differential Equations (Ordinary and Partial), Books and Allied (P) Ltd., Kolkata.
3. J G Chakravorty and P R Ghosh – Advanced Analytical Geometry, U N Dhur & Sons Pvt. Ltd, Kolkata.
4. R M Khan – Analytical Geometry of Two and Three Dimensions and Vector Analysis, New Central Book Agency (P) Ltd., Kolkata.
5. Arup Mukherjee and N K Bej – Analytical Geometry of two & Three Dimensions (Advanced Level), Books and Allied (P) Ltd., Kolkata.
6. S L Ross – Differential Equations, John Wiley & Sons, India.
7. M D Raisinghania – Ordinary and Partial Differential Equations, S. Chand, New Delhi.
8. V Sundarapandian – Ordinary and Partial Differential Equations, McGraw Hill Education (India) Pvt. Ltd., Chennai.
9. S Balachandra Rao and H R Anuradha – Differential Equations with Applications and Programs, Universities Press, Hyderabad.

<u>Question Pattern for Semester – 1 Examinations</u>

(Course Code: HMAT1CC02N & Paper: CC02)

Group – A (*Analytical Two Dimensional Geometry, 30 marks*)

(i) Answer **four** objective / MCQ type questions of 2 marks each from 5 given questions.

(ii) Answer **two** questions of 5 marks each from 4 given questions.

(iii) Answer **two** questions of 6 marks each from 4 given questions.

Group – B (*Differential Equations – I, 35 marks*)

(i) Answer **four** objective / MCQ type questions of 2 marks each from 5 given questions.

(ii) Answer **three** questions of 5 marks each from 5 given questions.

(iii) Answer **two** questions of 6 marks each from 4 given questions.

SEMESTER – 2		
Name of the Paper : **CC03**		
Name of the Course : Real Analysis – I		
Course Code : HMAT2CC03N		
Full Marks : 100		Credit : 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course a student will acquire knowledge

- on real number system from axiomatic stand point.
- on different facets of sequence and series of real numbers and the fundamental concept of their convergence through some of the most celebrated theorems of this area.
- and skill towards solving various problems related to the concepts of limit, continuity and differentiability of a real valued function of real variables. These cornerstone ideas will lay foundation for future development of real analysis as presented in CC07 of Semester – 3.

Detailed Syllabus

Real Analysis – I
[65 Marks]

1. **Real Number System**: Field Axioms. Concept of ordered field. Bounded set, *L.U.B.* (supremum) and *G.L.B.* (infimum) of a set. Properties of *L.U.B.* and *G.L.B.* of sum of two sets and scalar multiple of a set. Least upper bound axiom or completeness axiom. Characterization of $\mathbb{R}$ as a complete ordered field. Definition of an Archimedean ordered field. Archimedean property of $\mathbb{R}$. $\mathbb{Q}$ is Archimedean ordered field but not ordered complete. Linear continuum.
2. **Sets in** $\mathbb{R}$:
   (i) Intervals. Neighbourhood of a point. Interior point. Open set. Union, intersection of open sets. Every open set can be expressed as disjoint union of open intervals.
   (ii) Limit point and isolated point of a set. Criteria for *L.U.B.* and *G.L.B.* of a bounded set to be limit point of the set. Bolzano-Weierstrass theorem on limit point. Definition of derived set. Derived set of a bounded set A is contained in the closed interval $[\inf A, \sup A]$. Closed set. Complement of open set and closed set. Union and intersection of closed sets as a consequence. No nonempty proper subset of $\mathbb{R}$ is both open and closed.
   (iii) Dense set in $\mathbb{R}$ as a set having non-empty intersection with every open interval. $\mathbb{Q}$ and $\mathbb{R}\setminus\mathbb{Q}$ are dense in $\mathbb{R}$.
3. **Countability of sets**: Countability and uncountability of a set. Subset of a countable set is countable. Every infinite set has a countably infinite subset. Cartesian product of two countable sets is countable. $\mathbb{Q}$ is countable. Non-trivial intervals are uncountable. $\mathbb{R}$ is uncountable.

4. **Sequences of real numbers**:
   (i) Definition of a sequence as function from $\mathbb{N}$ to $\mathbb{R}$. Bounded sequence. Convergence (formalization of the concept of limit as an operation in $\mathbb{R}$) and non-convergence. Examples. Every convergent sequence is bounded and limit is unique. Algebra of limits.
   (ii) Sequential criterion of the limit point of a set in $\mathbb{R}$. Monotone sequences and their convergence. Sandwich rule. Nested interval theorem. Limit of some important sequences : $\left\{n^{\frac{1}{n}}\right\}_n$, $\{x^n\}_n$, $\left\{x^{\frac{1}{n}}\right\}_n$, $\{x_n\}_n$ with $\frac{x_{n+1}}{x_n} \to l$ and $|l| < 1$, $\left\{\left(1+\frac{1}{n}\right)^n\right\}_n$, $\left\{1+\frac{1}{1!}+\frac{1}{2!}+\cdots+\frac{1}{n!}\right\}_n$, $\{a^{x_n}\}_n (a > 0)$. Cauchy's first and second limit theorems.
   (iii) Subsequence. Subsequential limits, *lim sup* as the L.U.B. and *lim inf* as the G.L.B of a set containing all the subsequential limits. Alternative definition of *lim sup* and *lim inf* of a sequence using inequalities. A bounded sequence $\{x_n\}_n$ is convergent iff $lim\ sup\ x_n = lim\ inf\ x_n$. Every sequence has a monotone subsequence. Bolzano-Weierstrass' theorem. Cauchy's general principle of convergence.
5. **Limits and Continuity of real-valued functions of a real variable**:
   (i) Limit of a function at a point (the point must be a limit point of the domain set of the function). Sequential criteria for the existence of finite and infinite limit of a function at a point. Algebra of limits. Sandwich rule. Important limits like $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)$, $\lim_{x\to 0}\left(\frac{\log(1+x)}{x}\right)$, $\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)$ $(a > 0)$.
   (ii) Continuity of a function at a point. Continuity of a function on an interval and at an isolated point. Familiarity with the figures of some well-known functions :
$$y = x^a\left(a = 2,3,\frac{1}{2},-1\right), |x|, \sin x, \cos x, \tan x, \log x, e^x.$$
   Algebra of continuous functions as a consequence of algebra of limits. Continuity of composite functions. Examples of continuous functions. Continuity of a function at a point does not necessarily imply the continuity in some neighbourhood of that point.
   (iii) Bounded functions. Neighbourhood properties of continuous functions regarding boundedness and maintenance of same sign. Continuous function on $[a,b]$ is bounded and attains its bounds. Intermediate value theorem.
   (iv) Discontinuity of function, type of discontinuity. Step function. Piecewise continuity. Monotone function. Monotone function can have only jump discontinuity. Set of points of discontinuity of a monotone function is at most countable. Monotone bijective function from an interval to an interval is continuous and its inverse is also continuous.
   (v) Definition of uniform continuity and examples. Lipschitz condition and uniform continuity. Functions continuous on a closed and bounded interval is uniformly continuous. A necessary and sufficient condition under which a continuous function on a bounded open interval I will be uniformly continuous on I. A sufficient condition under which a continuous function on an unbounded open interval I will be uniformly continuous on I (statement only).
6. **Infinite Series of real numbers:**
   (i) Convergence, Cauchy's criterion of convergence.
   (ii) Series of non-negative real numbers: Tests of convergence – Cauchy's condensation test. Comparison test (ordinary form and upper limit and lower limit criteria), Ratio Test, Raabe's Test, Root test, Gauss's test (proof not required).
   (iii) Series of arbitrary terms: Absolute and conditional convergence.
   (iv) Alternating series: Leibnitz test.
   (v) Abel's and Dirichlet's test (statements and applications).

7. **Derivatives of real valued functions of a real variable:**
   (i) Definition of derivability. Meaning of sign of derivative. Chain rule.
   (ii) Successive derivative: Leibnitz theorem.
   (iii) Theorems on derivatives: Darboux theorem, Rolle's theorem, Mean value theorems of Lagrange and Cauchy – as an application of Rolle's Theorem. Taylor's theorem on closed and bounded interval with Lagrange's and Cauchy's form of remainder deduced from Lagrange's and Cauchy's mean value theorem respectively. Maclaurin's theorem as a consequence of Taylor's theorem. Statement of Maclaurin's Theorem on infinite series expansion. Expansion of e^x, $log\,(1+x)$, $(1+x)^m$, $sin\,x$, $cos\,x$ with their range of validity.
   (iv) Statement of L'Hospital's rule and its consequences. Point of local extremum (maximum, minimum) of a function in an interval. Sufficient condition for the existence of a local maximum/minimum of a function at a point (statement only). Determination of local extremum using first order derivative. Application of the principle of maximum/minimum in geometrical problems.

References:

1. Tom M Apostol – Mathematical Analysis, Narosa Publishing House
2. Tom M Apostol – Calculus (Vol I & II), John Wiley and Sons
3. W Rudin – Principles of Mathematical Analysis, McGraw-Hill (1976)
4. R G Bartle and D R Sherbert – Introduction to Real Analysis, John Wiley and Sons (Asia) Pvt. Ltd., Singapore (3rd Edition, 2002)
5. C C Pugh – Real Mathematical Analysis, Springer (2002)
6. Terence Tao – Analysis I, Hindustan Book Agency (2006)
7. Richard R Goldberg – Method of Real Analysis, John Wiley and Sons (1976)
8. S C Malik & S Arora – Mathematical Analysis, New Age International, New Delhi (4th Edition, 2014)

<u>Question Pattern for Semester – 2 Examinations</u>

(Course Code: HMAT2CC03N & Paper: CC03)

(i) Answer **eight** objective / MCQ type questions of 2 marks each from 9 given questions.

(ii) Answer **three** questions of 3 marks each from 5 given questions.

(iii) Answer **four** questions of 10 marks each from 6 given questions. Each may contain further parts.

SEMESTER – 2		
Name of the Paper : **CC04**		
Name of the Course : Abstract Algebra – II, Linear Algebra – I		
Course Code : HMAT2CC04N		
Full Marks : 100		Credit : 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course a student will acquire

- knowledge on deeper structural issues related to group theory in continuation with their earlier exposure in CC01 of Semester – 1.
- elementary knowledge on different algebraic structures of double composition and their interrelations.
- knowledge and skill towards solving problems on matrix theory, which is of paramount importance as a tool, for their future courses CC06 [Semester – 3] and CC09 [Semester – 4] on Linear Algebra, a subject necessary for almost all allied science subjects.

Detailed Syllabus

Group – A [35 Marks]
(Abstract Algebra – II)

1. **Groups**: Homomorphism and Isomorphism of Groups. Kernel of a Homomorphism. First Isomorphism Theorem. Classification of finite and infinite cyclic groups. Cayley's theorem, properties of isomorphisms. Second and Third isomorphism theorems. Direct product of groups (basic ideas and simple applications).
2. **Rings and Fields**: Definition and examples of rings, properties of rings, subrings, necessary and sufficient condition for a non-empty subset of a ring to be a subring, integral domains and fields, subfield, necessary and sufficient condition for a nonempty subset of a field to be a subfield, characteristic of a ring. Ideal, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals. Ring homomorphisms, properties of ring homomorphisms. First, second and third isomorphism theorems on ring, Correspondence theorem.

Group – B [30 Marks]
(Linear Algebra – I)

1. Matrices of real and complex numbers; algebra of matrices, symmetric and skew-symmetric matrices, Hermitian and skew-Hermitian matrices, orthogonal and unitary matrices.
2. $n \times n$ Determinants, Laplace expansion, Vandermonde's determinant. Symmetric and skew-symmetric determinants (no proof of theorems required, problems on determinants

up to order 4). Adjoint of a square matrix. For a square matrix A, $A.adj\,A = adj\,A.A = (det A)I_n$. Invertible matrix, non-singularity. Inverse of an orthogonal matrix. Jacobi's 1st and 2nd theorems on determinants and its applications.

3. Elementary operations on matrices. Echelon matrix. Rank of a matrix. Determination of rank of a matrix (relevant results are to be stated only). Elementary matrices, fully reduced Normal form. Rank factorization. Evaluation of determinant by Gaussian elimination. Triangular factorization $A = LU, A = LDV, PA = LU$ and $EA = R$.
4. Congruence of matrices – statement and application of relevant results, Normal form of a matrix under congruence.

References:

1. M K Sen, S Ghosh, P Mukhopadhyay and S Maity – Topics in Abstract Algebra, University Press, Hyderabad (3rd Edition, 2020)
2. D S Malik, John M Mordeson and M K Sen – Fundamentals of Abstract Algebra, McGraw Hill, New York (1997)
3. John B Fraleigh – A First Course in Abstract Algebra, Pearson (7th Edition, 2002)
4. M Artin – Abstract Algebra, Pearson (2nd Edition, 2011)
5. Joseph A Gallian – Contemporary Abstract Algebra, Narosa Publishing House, New Delhi (4th Edition, 1999)
6. Joseph J Rotman – An Introduction to the Theory of Groups, Springer (4th Edition, 1995)
7. I N Herstein – Topics in Algebra, Wiley Eastern Limited, India (1975)
8. Kenneth Hoffman and Ray Alden Kunze – Linear Algebra, Prentice-Hall of India Pvt. Ltd. (2nd Edition, 1971)
9. Stephen H Friedberg, Arnold J Insel and Lawrence E Spence – Linear Algebra, Prentice-Hall of India Pvt. Ltd., New Delhi (4th Edition, 2004)
10. Gilbert Strang – Linear Algebra and its Applications, Thomson (2007)
11. Gilbert Strang – Fundamentals of Linear Algebra, Cambridge University Press (2007)
12. S Lang – Introduction to Linear Algebra, Springer (2nd Edition, 2005)
13. S Kumaresan – Linear Algebra – A Geometric Approach, Prentice Hall of India (1999)

Question Pattern for Semester – 2 Examinations

(Course Code: HMAT2CC04N & Paper: CC04)

Group – A (Abstract Algebra – II, 35 marks)

i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

ii) Answer **five** questions of 5 marks each from 7 given questions. Each question may contain further parts.

Group – B (Linear Algebra – I, 30 marks)

i) Answer **three** objective / MCQ type questions of 2 marks each from 4 given questions.

ii) Answer **three** questions of 8 marks each from 5 given questions. Each may contain further parts.

SEMESTER – 3		
Name of the Paper : **CC05**		
Name of the Course : Analytical Three Dimensional Geometry and Vector Algebra, Multivariate Calculus – I		
Course Code : HMAT3CC05N		
Full Marks : 100		Credit : 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course a student will acquire knowledge

- on three dimensional analytical geometry (only Cartesian coordinate system) and will be able to solve problems related to those topics, as a continuation of their previous concepts from plus two level. Also acquire general ideas on various curvilinear coordinate systems.
- and skill towards solving various problems related to vector algebra which has useful applications in various branches of Mathematics and Physics, as a continuation of their previous concepts from plus two level.
- and skill towards solving various problems related to multivariate calculus, which is a powerful tool for understanding the geometry of real n-dimensional space.

Detailed Syllabus

Group – A (30 Marks)

[Analytical Three Dimensional Geometry and Vector Algebra]

1. (a) Revision on plane and straight line in 3D.
   (b) Transformation of rectangular axes by translation, rotation and their combination.
2. Sphere: General equation, Circle, Sphere through a circle, sphere through the intersection of two spheres. Radical plane. Tangent line, tangent plane and normal.
3. Cone: Right circular cone. General homogeneous second degree equation represents a cone. Section of a cone by a plane as a conic and as a pair of straight lines. Condition for three perpendicular generators. Reciprocal cone.
4. Cylinder: General equation of a cone. Generators parallel to either of the axes. Right circular cylinder.
5. Ellipsoid, Hyperboloid, Paraboloid: Canonical equations only.
6. General quadric surface: Tangent plane, Normal, Enveloping cone and enveloping cylinder.
7. Ruled surface. Generating lines of hyperboloid of one sheet and hyperbolic paraboloid.
8. Introduction to curvilinear coordinate system.
9. Vector triple product. Product of four vectors. Reciprocal vectors. Application to geometrical problems.

Group – B (35 Marks)

[Multivariate Calculus – I]

1. Concept of neighbourhood of a point in $\mathbb{R}^n (n > 1)$, interior point, limit point, open set and closed set in $\mathbb{R}^n (n > 1)$.
2. Functions from $\mathbb{R}^n (n > 1)$ to $\mathbb{R}^m (m \geq 1)$, limit and continuity of functions of two or more variables. Partial derivatives, total derivative and differentiability. Sufficient condition for differentiability. Higher order partial derivatives and theorems on equality of mixed partial derivatives for a function of two variables. Chain rule for one and two independent parameters. Euler's theorems and its converse. Directional derivatives, the gradient, maximal and normal property of the gradient, tangent planes.
3. Jacobian of two and three variables, simple properties including function dependence. Concept of Implicit function. Statement and simple application of implicit function theorem for two variables. Differentiation of Implicit function.
4. Taylor's theorem for functions two variables. Extreme points and values of functions of two variables. Lagrange's method of undetermined multipliers for function of two variables (problems only). Constrained optimization problems.

References:

1. J G Chakravorty and P R Ghosh – Advanced Analytical Geometry, U N Dhur & Sons Pvt. Ltd, Kolkata.
2. R M Khan – Analytical Geometry of Two and Three Dimensions and Vector Analysis, New Central Book Agency (P) Ltd., Kolkata.
3. Arup Mukherjee and N K Bej – Analytical Geometry of two & Three Dimensions (Advanced Level), Books and Allied (P) Ltd., Kolkata.
4. S C Mallik & S Arora – Mathematical Analysis, New Age International, New Delhi.
5. E Marsden, A J Tromba and A Weinstein – Basic Multivariable Calculus, Springer.

Question Pattern for Semester – 3 Examinations

(Course Code: HMAT3CC05N & Paper: CC05)

Group – A (Analytical Three Dimensional Geometry and Vector Algebra, 30 marks)

i) Answer **three** objective / MCQ type questions of 2 marks each from 4 given questions.

ii) Answer **two** questions of 8 marks each from 4 given questions from Article 1 to 8. Each question may contain further parts.

iii) Answer **one** question of 8 marks each from 2 given questions from Article 9. Each question may contain further parts.

Group – B (Multivariate Calculus – I, 35 marks)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

(ii) Answer **five** questions of 5 marks each from 7 given questions. Each question may contain further parts.

SEMESTER – 3	
Name of the Paper : **CC06**	
Name of the Course : Linear Algebra – II, Application of Calculus	
Course Code : HMAT3CC06N	
Full Marks : 100	Credit: 6
Theory	Credits : 5 Full Marks : 65+20*
Tutorial	Credits : 1 Full Marks : 15
Number of classes required : 100	
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance	

Course Outcomes: At the end of studying this course a student

- will acquire knowledge on Vector Space theory that will allow them to appreciate Linear algebra as a tool for learning Geometry of higher dimensional spaces through the language of Algebra. They will also be able to solve problems related to matrix theory up to orthogonalization. This will be continued further in CC09 of Semester – 4.
- will be geared up towards appreciating the mathematical theory behind the Linear Programming problems to be taught in DSE3 of Semester – 6.
- will acquire skills towards solving various problems on Geometry through the powerful tools of Differential and Integral Calculus which is of paramount importance in Mathematics and Physics.

Detailed Syllabus

Group – A [30 Marks]
(Linear Algebra – II)

1. Vector / Linear space: Definitions and examples, Subspace, Union and intersection of subspaces. Linear sum of two subspaces. Linear combination, independence and dependence, linear span. Generators of vector space, finite dimensional real and complex vector space, basis. Deletion theorem, Extension theorem and Replacement Theorem. Dimension of a vector space. Extraction of basis. Vector space over a finite field.
2. Row space, column space, null space and left null space of a matrix. Row rank and column rank of matrix. Equality of row rank, column rank and rank of a matrix. Fundamental theorem of Linear algebra (Part I and Part II). Every matrix transforms its row space into column space. Linear homogeneous system of equations: Solution space, related results using idea of rank, linear non-homogeneous system of equations – solvability and solution by Gauss-Jordan elimination. Free and basic variables, pivots.
3. Inner Product Space: Norm, Euclidean and Unitary Vector Space, Definition and examples, Triangle inequality and Cauchy-Schwarz Inequality, Orthogonality of vectors, Orthonormal basis, Gram-Schmidt Process of orthonomalization, orthogonal complement.

Group – B [35 Marks]
(Application of Calculus)

1. **Applications of Differential Calculus** :
   - (i) Tangents and normals: Sub-tangent and sub-normal. Angle of intersection of curves. Pedal equation of a curve, pedal of a curve.
   - (ii) Curvature: Radius of curvature, centre of curvature, chord of curvature, evolute of a curve.
   - (iii) Rectilinear Asymptotes (Cartesian, polar and parametric curve).
   - (iv) Envelope of family of straight lines and of curves (problems only).
   - (v) Concavity, convexity, singular points, nodes, cusps, points of inflexion, simple problems on species of cusps of a curve (Cartesian curves only).
2. **Applications of Integral Calculus**:
   - (i) Area enclosed by a curve.
   - (ii) Determination of C.G
   - (iii) Moments and products of inertia (Simple problems only).
3. Familiarity with the figure of following curves: Periodic curves with suitable scaling, Cycloid, Catenary, Lemniscate of Bernoulli, Astroid, Cardiode, Folium of Descartes, equiangular spiral.

References:

1. Kenneth Hoffman and Ray Alden Kunze – Linear Algebra, Prentice-Hall of India Pvt. Ltd. (2^{nd} Edition, 1971)
2. Stephen H Friedberg, Arnold J Insel and Lawrence E. Spence – Linear Algebra, Prentice-Hall of India Pvt. Ltd., New Delhi (4^{th} Edition, 2004)
3. Gilbert Strang – Linear Algebra and its Applications, Thomson (2007)
4. Gilbert Strang – Fundamentals of Linear Algebra, Cambridge University Press (2007)
5. S Lang – Introduction to Linear Algebra, Springer (2^{nd} Edition, 2005)
6. S Kumaresan – Linear Algebra -A Geometric Approach, Prentice Hall of India (1999)
7. D Sengupta – Application of Calculus, Books & Allied (P) Ltd, Kolkata (2013)
8. G.B. Thomas and R.L. Finney, Calculus, 9th Ed., Pearson Education, Delhi, 2005.
9. M. J. Strauss, G.L. Bradley and K. J. Smith, Calculus, 3rd Ed., Dorling Kindersley (India) P. Ltd. (Pearson Education), Delhi, 2007
10. S Bandyopadhyay and S K Maity – Application of Calculus, Theory and Problems – Academic Publishers
11. R K Ghosh and K C Maity – Differential Calculus – New Central Book Agency

Question Pattern for Semester – 3 Examinations

(Course Code: HMAT3CC06N & Paper: CC06)

Group – A (Linear Algebra – II, 30 marks)

(i) Answer **three** objective / MCQ type questions of 2 marks each from 4 given questions.

(ii) Answer **three** questions of 8 marks each from 5 given questions. Each question may contain further parts.

Group – B (Application of Calculus, 35 marks)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

(ii) Answer **three** questions of 5 marks each from 5 given questions from Article – 1. Each question may contain further parts.

(iii) Answer **two** questions of 5 marks each from 4 given questions from Article - 2. Each question may contain further parts.

SEMESTER – 3		
Name of the Paper : **CC07**		
Name of the Course : Real Analysis – II		
Course Code : HMAT3CC07N		
Full Marks : 100		Credit : 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course which is a continuation of CC03 of Semester – 2, a student will acquire

- knowledge on various ideas of compactness related to real number system.
- knowledge on different facets of the celebrated concept of Riemann Integration, which is a generalization of their earlier knowledge of Newtonian Integration done during plus two level.
- working knowledge and skill towards solving various problems related to the series and sequence of functions and power series.

Detailed Syllabus

Real analysis – II [65 Marks]

1. Compactness in $\mathbb{R}$: Open cover of a set. Compact set in $\mathbb{R}$, a set is compact iff it is closed and bounded. Sequential compactness, Frechét compactness. Compactness and continuity.
2. Connectedness in $\mathbb{R}$: Definition and examples, connected subsets of $\mathbb{R}$, path-connectedness, Continuity and Connectedness.
3. Partition and refinement of partition of a closed and bounded interval. Upper Darboux sum $U(P, f)$ and lower Darboux sum $L(P, f)$ and associated results. Upper integral and lower integral. Darboux's theorem. Darboux's definition of integration over a closed and bounded interval. Riemann's definition of integrability. Equivalence with Darboux definition of integrability (statement only). Necessary and sufficient condition for Riemann integrability.
4. Concept of negligible set (or set of measure zero). Examples of negligible sets: any subset of a negligible set, finite set, countable union of negligible sets. Lebesgue-Vitalli theorem on Riemann integration. Example of Riemann integrable functions.
5. Integrability of sum, scalar multiple, product, quotient, modulus of Riemann integrable functions. Properties of Riemann integrable functions arising from the above results.
6. Function defined by definite integral and its properties. Anti-derivative (primitive or indefinite integral). Properties of Logarithmic function defined as the definite integral $\int_1^x \frac{dt}{t}, x > 0$.
7. Fundamental theorem of Integral Calculus. First Mean Value theorem of integral calculus. Statement and applications of Second Mean Value theorem of integral calculus.
8. Sequence of functions, Series of functions, Power series.

References:

1. Tom M Apostol – Mathematical Analysis, Narosa Publishing House
2. Tom M Apostol – Calculus (Vol I & II), John Wiley and Sons
3. W Rudin – Principles of Mathematical Analysis, McGraw-Hill (1976)
4. R G Bartle and D. R. Sherbert – Introduction to Real Analysis, John Wiley and Sons (Asia) Pvt. Ltd., Singapore (3^{rd} Edition, 2002)
5. CC Pugh – Real Mathematical Analysis, Springer (2002)
6. Terence Tao – Analysis I, Hindustan Book Agency (2006)
7. Richard R Goldberg – Method of Real Analysis, John Wiley and Sons (1976)
8. S C Mallik& S Arora – Mathematical Analysis, New Age International, New Delhi (2014)

Question Pattern for Semester – 3 Examinations

(Course Code: HMAT3CC07N and Paper: CC07)

(i) Answer **nine** objective / MCQ type questions of 2 marks each from 10 given questions.

(ii) Answer **one** question of 7 marks from 2 given questions from Articles – 1 and 2. Each question may contain further parts.

(iii) Answer **five** questions of 5 marks each from 7 given questions from Articles – 3 to 7. Each question may contain further parts.

(iv) Answer **three** questions of 5 marks each from 4 given questions from Article – 8. Each question may contain further parts.

SEMESTER – 4	
Name of the Paper : **CC08**	
Name of the Course : Analytical Mechanics: Analytical Dynamics, Analytical Statics	
Course Code : HMAT4CC08N	
Full Marks : 100	Credit: 6
Theory	Credits : 5 Full Marks : 65+20*
Tutorial	Credits : 1 Full Marks : 15
Number of classes required : 100	
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance	

Analytical Mechanics

Course Outcomes: At the end of studying this course a student will acquire

- knowledge of rectilinear and planar motion of a particle
- knowledge and skill for solving problems on Analytical Statics related to coplanar forces.
- knowledge and skill for solving problems on Analytical Statics related to three dimensional forces.

Detailed Syllabus

Group – A (40 Marks)
[Analytical Dynamics]

1. Newton's laws of motion. Inertial and non-inertial frames of reference (concepts only).
2. Rectilinear motion under variable forces. Simple harmonic motion. Motion of elastic strings and springs. Damped harmonic oscillator. Forced oscillations. Vertical motion under gravity when the resistance of the atmosphere varies as some integral power of the velocity.
3. Velocity and acceleration in plane Cartesian and polar coordinates. Motion of a projectile in a resisting medium.
4. Motion in a rotating frame of reference. Centrifugal and Coriolis forces. Euler Force.
5. Tangential and normal accelerations. Motion of a simple pendulum when the amplitude is not small. Constrained motion of a particle on smooth and rough curves.
6. Motion of a system of particles. Linear and angular momentum of a system of particles. Energy of a system of particles. Conservation laws.
7. Motion when the mass varies.
8. Central forces and central orbits. Motion under inverse square law of attraction. Kepler's laws of planetary motion. Stability of nearly circular orbits. Slightly disturbed orbits. Motion of artificial satellites.

Group – B (25 Marks)
[Analytical Statics]

1. **Coplanar Forces**: Astatic Equilibrium, Astatic Centre, Positions of equilibrium of a particle lying on a smooth plane curve under action of given force. Action at a joint in a frame work.
2. **Friction**: Laws of Friction, Angle of friction, Cone of friction. Positions of equilibrium of a particle lying on a rough plane curve, rough space curve and rough surface under the action of any given forces.
3. **Virtual work**: Workless constraints - examples, virtual displacements and virtual work. The principle of virtual work. Deductions of the necessary and sufficient conditions of equilibrium of an arbitrary force system in plane and space, acting on a rigid body.
4. **Stability of equilibrium**: Conservative force field, energy test of stability, condition of stability of a perfectly rough heavy body lying on a fixed body. Rocking stones.
5. **Forces in the three dimensions**: Moment of a force about a line. Axis of a couple. Resultant of any two couples acting on a body. Resultant of any number of couples acting on a rigid body. Reduction of a system of forces acting on a rigid body. Resultant force is an invariant of the system but the resultant couple is not an invariant. Conditions of equilibrium of a system of forces acting on a body. Deductions of the conditions of equilibrium of a system of forces acting on a rigid body from the principle of virtual work. Poinsot's central axis. A given system of forces can have only one central axis. Wrench, Pitch, Intensity and Screw. Condition that a given system of forces may have a single resultant. Invariants of a given system of forces. Equations of the central axis of a given system of forces.

References:

1. S L Loney – An Elementary Treatise on the Dynamics of a Particle and of Rigid Bodies, Cambridge University Press.
2. D T Greenwood – Principle of Dynamics, PHI, New Delhi
3. S L Loney – An Elementary Treatise on Statics, Cambridge University Press.
4. M C Ghosh – Analytical Statics, Shreedhar Prakashani, Kolkata.
5. S A Mollah – Analytical Statics, Books & Allied (P) Ltd, Kolkata.
6. S Pradhan & S Sinha – Analytical Statics, Academic Publishers, Kolkata.

Question Pattern for Semester – 4 Examinations

(Course Code: HMAT4CC08N & Paper: CC08)

Group – A (Analytical Dynamics, 40 marks)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

(ii) Answer **five** questions of 6 marks each from 7 given questions. Each question may contain further parts.

Group – B (Analytical Statics, 25 marks)

(i) Answer **two** objective / MCQ type questions of 2 marks each from 4 given questions.

(ii) Answer **one** question of 3 marks from 2 given questions.

(iii) Answer **two** questions of 6 marks each from 4 given questions from Article 1 to 4. Each may contain further parts.

(iv) Answer **one** question of 6 marks from 2 given questions from Article 5. Each may contain further parts.

<table>
<tr><td colspan="3" align="center">SEMESTER – 4</td></tr>
<tr><td colspan="3">Name of the Paper : CC09</td></tr>
<tr><td colspan="3">Name of the Course : Linear Algebra – III</td></tr>
<tr><td colspan="3">Course Code : HMAT4CC09N</td></tr>
<tr><td colspan="2">Full Marks : 100</td><td>Credit: 6</td></tr>
<tr><td>Theory</td><td colspan="2">Credits : 5
Full Marks : 65+20*</td></tr>
<tr><td>Tutorial</td><td colspan="2">Credits : 1
Full Marks : 15</td></tr>
<tr><td colspan="3">Number of classes required : 100</td></tr>
<tr><td colspan="3">*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance</td></tr>
</table>

Course Outcomes: At the end of studying this course, which is a continuation of CC06 of Semester – 3, a student will acquire knowledge on

- the interplay of the theory of Vector Space and that of the Matrix, that will allow them to appreciate Linear Transformation among vector spaces and matrices to be the two sides of the same coin.
- Vector spaces over the field of Complex number, towards a better appreciation of the power of matrix theory.
- Eigen values and (orthogonal) Eigen vectors of a matrix or a linear transformation that will allow them to understand the abstract coordinatization of higher dimensional spaces through spectral resolution for studying its geometry. These ideas and related analysis will be continued further in a DSE paper of a later semester.

Detailed Syllabus

Linear Algebra – III [65 Marks]

1. Linear transformation on vector space: Definition, null space, range, rank and nullity, rank-nullity theorem, simple applications, non-singular linear transformation, inverse of linear transformation. A $m \times n$ real matrix as a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. Linear Transformation in $\mathbb{R}^2$, matrices of rotation, matrices of projection and reflection on θ-line.

2. Natural isomorphism $\varphi : V \to \mathbb{R}^n$ (for an n-dimensional vector space V) and co-ordinate vector $[x]_\alpha$ with respect to an ordered basis α of V. Matrices of linear transformations: $T: V_\alpha \to W_\beta$ corresponds to unique $[T]_\alpha^\beta$ such that $[T(x)]_\beta = [T]_\alpha^\beta [x]_\alpha$ [finite dimensional cases]. Looking back to some simple matrix properties in the light of linear transformation: product of two matrices, rank of a matrix and inverse of matrix. Change of basis, similarity of matrices.

3. Orthogonal projection and least square, $A = QR$ factorization,

4. Characteristic and Minimal Polynomial of a matrix, their interrelation, Eigen vectors of a matrix and those of a linear operator. Cayley-Hamilton theorem, properties of Eigen values and Eigen vectors. Eigen space of a linear operator, Leverrier-Fadeev method for Eigen vectors of a matrix, Eigen values of Circulant matrix and corresponding Eigen vector and its relation with Vandermonde determinant.

5. Diagonalizability and Diagonalization of a matrix or a linear operator (statement and application of relevant results). Looking back at geometry of real and complex Eigen values and Eigen vectors. Orthogonal diagonalization of symmetric matrix. Some Applications.

6. Dual spaces, dual basis, adjoint transformation and transpose of a matrix. Invariant subspace, Annihilator of a subspace.

7. Normal and self-adjoint operators, unitary and orthogonal operators, Triangularization, Schur's lemma and Schur decomposition, Unitary diagonalization of a Normal matrix, Spectral theorem of normal operator (matrix). Applications, Spectral Resolution of a diagonalizable operator using Minimal Polynomial.

8. Real Quadratic Form involving three variables. Reduction to Normal Form under Congruence and also by orthogonal transformation of variables (Statements of relevant theorems and applications). Rank, signature and Index of a real quadratic form. Classification of Real Quadratic forms. Relation with eigenvalues of the associated symmetric matrix.

References:

1. Kenneth Hoffman and Ray Alden Kunze – Linear Algebra, Prentice-Hall of India Pvt. Ltd. (2nd Edition, 1971)
2. Stephen H Friedberg, Arnold J Insel and Lawrence E Spence – Linear Algebra, Prentice- Hall of India Pvt. Ltd., New Delhi (4th Edition, 2004)
3. Gilbert Strang – Linear Algebra and its Applications, Thomson (2007)
4. Gilbert Strang – Fundamentals of Linear Algebra, Cambridge University Press (2007)
5. S Lang – Introduction to Linear Algebra, Springer (2nd Edition, 2005)
6. S Kumaresan – Linear Algebra-A Geometric Approach, Prentice Hall of India (1999)
7. Michael Taylor – Linear Algebra, Springer (3rd Edition, 2015)
8. Jin Ho Kwak and S Hong – Linear Algebra, Springer (2004)
9. F Zhang – Matrix Theory, Springer (2011)

Question Pattern for Semester – 4 Examinations

(Course Code: HMAT4CC09N & Paper – CC09)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 8 given questions.

(ii) Answer **five** questions of 11 marks each. These questions may either be set to be chosen straight out of 7 given questions, or they may be clubbed to be chosen as one of the two alternatives given. Each question may contain further parts.

<table>
<tr><td colspan="3" align="center">SEMESTER – 4</td></tr>
<tr><td colspan="3">Name of the Paper: CC10</td></tr>
<tr><td colspan="3">Name of the Course : Multivariate Calculus – II (Including vector calculus): Multiple Integral and Vector Calculus, Complex Analysis</td></tr>
<tr><td colspan="3">Course Code : HMAT4CC10N</td></tr>
<tr><td colspan="2">Full Marks : 100</td><td>Credit: 6</td></tr>
<tr><td>Theory</td><td colspan="2">Credits : 5
Full Marks : 65+20*</td></tr>
<tr><td>Tutorial</td><td colspan="2">Credits : 1
Full Marks : 15</td></tr>
<tr><td colspan="3">Number of classes required : 100</td></tr>
<tr><td colspan="3">*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance</td></tr>
</table>

Multivariate Calculus – II (Including vector calculus)

Course Outcomes: At the end of studying this course a student will acquire

- elementary knowledge and skill of solving problems on multiple integral and centre of gravity.
- knowledge on vector calculus and their applications in Mathematical Physics.
- some of the elementary but fundamental knowledge of Complex analysis.

Detailed Syllabus

Group – A (35 Marks)

[Multiple integral and Vector Calculus]

1. **Multiple integral**: Concept of upper sum, lower sum, upper integral, lower-integral and double integral (no rigorous treatment is needed). Statement of existence theorem for continuous functions. Iterated or repeated integral, change of order of integration. Triple integral. Cylindrical and spherical coordinates. Change of variables in double integrals and triple integrals. Transformation of double and triple integrals (problems only). Determination of volume and surface area by multiple integrals (problems only).
2. **Centre of Gravity**: General formula for the determination of C.G. Determination of position of C.G. of any arc, area of solid of known shape by method of integration.
3. **Vector Calculus:** Vector function of a single real variable. Derivative of a vector.
4. Concept of scalar and vector fields. Gradient of a scalar field and its physical significance. Divergence and Curl of a vector field and their physical significance.
5. Line and surface integrals of vectors. Conservative vector field. Statement and verification of Green's theorem, Stokes' theorem and the divergence theorem.

Group – B (30 Marks)

[Complex Analysis]

1. Stereographic projection. Regions in the complex plane. Limits, limits involving the point at infinity. Continuity of functions of complex variable.
2. Derivatives, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability. Analytic functions, exponential function, logarithmic function, trigonometric functions and hyperbolic functions. Möbius transformation.
3. Power series: Cauchy-Hadamard theorem. Determination of radius of convergence. Uniform and absolute convergence of power series. Analytic functions represented by power series. Uniqueness of power series.
4. Contours, complex integration along a contour and its examples, upper bounds for moduli of contour integrals. Cauchy-Goursat theorem (statement only) and its consequences, Cauchy integral formula.

References:

1. S C Mallik & S Arora – Mathematical Analysis, New Age International, New Delhi.
2. J E Marsden, A J Tromba, A Weinstein – Basic Multivariable Calculus – Springer
3. M R Spiegel – Schaum’s outline of Vector Analysis.
4. P K Nayak – Vector Algebra and Analysis with Applications, Universities Press, Hyderabad.
5. A A Shaikh – Vector Analysis with Applications, Narosa Publishing House, New Delhi.
6. James Ward Brown and R V Churchill – Complex Variables and Applications, McGraw Hill.
7. S Ponnusamy – Foundations of Complex Analysis, Springer.
8. Lars Ahlfors – Complex Analysis, McGraw Hill.

Question Pattern for Semester – 4 Examinations

(Course Code: HMAT4CC10N & Paper: CC10)

Group – A (Multiple integral and Vector Calculus, 35 marks)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

(ii) Answer **one** question of 5 marks from 2 given questions from Article 1. Each question may contain further parts.

(iii) Answer **one** question of 5 marks from 2 given questions from Article 2. Each question may contain further parts.

(iv) Answer **three** questions of 5 marks each from 4 given questions from Article 3 to 5. Each question may contain further parts.

Group – B (Complex Analysis, 30 marks)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

(ii) Answer **four** questions of 5 marks each from 6 given questions. Each may contain further parts.

SEMESTER – 5		
Name of the Paper : **CC11**		
Name of the Course: Real Analysis – III, Differential Equations – II (Including PDE)		
Course Code : HMAT5CC11N		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course, which is a continuation of CC07 of Semester – 3, a student

- will acquire knowledge on various types of improper integral and their convergence, rudimentary knowledge and working skill on Fourier Series representation of functions, which is one of the pivotal concepts of mathematics as a whole.
- will acquire knowledge on linear dynamical system.
- will acquire elementary knowledge and skill of solving problems on certain types of linear and non-linear partial differential equations, also acquire knowledge on certain types of second order partial differential equations and their applications in Mathematical Physics.

Detailed Syllabus

Group – A (20 Marks)

[Real Analysis – III]

1. **Improper integral**:
   (i) Range of integration- finite or infinite. Necessary and sufficient condition for convergence of improper integral in both cases.
   (ii) Tests of convergence: Comparison and μ-test. Absolute and non-absolute convergence and inter-relations. Statement of Abel's and Dirichlet's test for convergence on the integral of a product.
   (iii) Convergence and properties of Beta and Gamma function.
2. **Fourier series**
   Trigonometric series. Statement of sufficient condition for a trigonometric series to be a Fourier series. Fourier coefficients for periodic functions defined on $[-\pi, \pi]$. Statement of Dirichlet's condition of convergence. Statement of theorem of sum of Fourier series.
3. Differentiation under the integral sign, Leibniz's rule (problems only).

Group – B (45 Marks)

[Differential Equations – II (Including PDE)]

1. **Linear Dynamical Systems**: Definition of a dynamical system. Linear homogeneous plane autonomous systems. Concept of Poincare's phase plane and phase portraits. Fixed or critical points. Solving initial value problems by eigenvalue-eigenvector method. Nature of phase portraits for different types of eigenvalues. Classification of critical points.
2. **Partial differential equations of the first order**: Lagrange's method of solution. Non-linear first order partial differential equations. Charpit's general method of solution and some special types of equations which can be solved easily by methods other than the general method.
3. **Partial differential equations of the second order**: Derivation of heat equation, wave equation and Laplace equation. Classification of second order linear equations as hyperbolic, parabolic or elliptic. Reduction of second order linear equations to canonical forms.
4. **Cauchy problem**: Cauchy-Kowalewskaya theorem (statement only). Cauchy problem of finite and infinite string. Initial and boundary value problems. Semi-infinite string problem. Equations with non-homogeneous boundary conditions. Non-homogeneous wave equation. Method of separation of variables: solution of the vibrating string problem and the heat conduction problem.

References:

1. Tom M Apostol – Mathematical Analysis, Narosa Publishing House, New Delhi.
2. Tom M Apostol – Calculus (Vol I & II), John Wiley and Sons.
3. W Rudin – Principles of Mathematical Analysis, McGraw-Hill.
4. R G Bartle and D R Sherbert – Introduction to Real Analysis, John Wiley and Sons (Asia) Pvt. Ltd., Singapore.
5. C C Pugh – Real Mathematical Analysis, Springer.
6. Terence Tao – Analysis I, Hindustan Book Agency.
7. Richard R Goldberg – Method of Real Analysis, John Wiley and Sons.
8. S C Mallik & S Arora – Mathematical Analysis, New Age International, New Delhi.
9. N Mandal and B Pal – Differential Equations (Ordinary and Partial), Books & Allied (P) Ltd., Kolkata
10. Ian N Sneddon – Elements of Partial Differential equations, Mcgraw-Hill International Edition.
11. K Sankara Rao – Introduction to Partial Differential Equations, PHI, New Delhi.
12. Dipak K Ghosh – Introduction to Partial Differential Equation and Laplace Transform, New Central Book Agency (P) Ltd., Kolkata.

<u>Question Pattern for Semester – 5 Examinations</u>

(Course Code: HMAT5CC11N & Paper: CC11)

Group – A (Real Analysis – III, 20 marks)

(i) Answer **two** objective / MCQ type questions of 2 marks each from 3 given questions.

(ii) Answer **one** question of 8 marks from 2 given questions from Article 1. Each question may contain further parts

(iii) Answer **one** question of 8 marks from 2 given questions from Article 2 & 3. Each question may contain further parts

Group – B (Differential Equations – II [Including PDE], 45 marks)

(iii) Answer **four** objective / MCQ type questions of 2 marks each from 5 given questions.

(iv) Answer **one** question of 3 marks from 2 given questions.

(v) Answer **two** questions of 5 marks each from 3 given questions from Article 1. Each question may contain further parts.

(vi) Answer **four** questions of 6 marks each from 6 given questions from Article 2 to 4. Each question may contain further parts.

SEMESTER – 5		
Name of the Paper : **CC12**		
Name of the Course : General Topology		
Course Code : HMAT5CC12N		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this paper a student

- will acquire knowledge on various aspects of the theory of Metric spaces, which is a generalization of their previous knowledge on Real Analysis.
- will acquire fundamental knowledge on general topological spaces as a continuation and generalization of their knowledge of real, complex analysis and metric spaces.
- will acquire very important knowledge that will help the students to solve problems in various national entrance examinations towards their next level of study.

Detailed Syllabus

General Topology (65 marks)

1. Metric Spaces: Basic concepts, convergence and continuity, completeness and total boundedness, compactness, sequential compactness, limit point compactness, Bolzano-Weierstrass property. Connectedness. Arzela Ascoli Theorem (Statement with applications).
2. Topological spaces: Definition of Topological spaces, closed sets, interior points, boundary points, isolated points, limit point, dense set, nowhere dense set, first category and second category set, Baire's category theorem, convergence and continuity, homeomorphisms, base and subbase, weak topology and product topology, countability axioms, separation axioms T1-T5, compactness, connectedness, path connectedness, total disconnectedness.

References:

1. S Kumaresan – Topology of Metric Spaces, Narosa Publishing House (2nd Edition, 2011)
2. PK Jain and K Ahmad – Metric Spaces, Narosa Publishing House.
3. Satish Shirali and Harikishan L Vasudeva – Metric Spaces, Springer (2006).
4. Manabendra Nath Mukherjee – Elements of Metric Spaces, Academic Publishers, Kolkata (4th Edition, 2015)
5. J R Munkres – Topology - A First Course, Prentice Hall of India Pvt. Ltd (2000)
6. G F Simmons – Introduction to Topology and Modern Analysis, McGraw Hill (1963)
7. Introduction to General Topology; K.D. Joshi; New Age International (P) Limited, Publishers, New Delhi (2004)
8. J Dugundji – Topology, Allyn and Bacon (1966)

Question Pattern for Semester – 5 Examinations

(Course Code: HMAT5CC12N and Paper: CC12)

(i) Answer **five** objective / MCQ-type questions of 2 marks each from six given questions.

(ii) Answer **four** questions of 5 marks each from six given questions from Article 1. Each question may contain further parts.

(iii) Answer **five** questions of 7 marks each from seven given questions from Article 2. Each question may contain further parts.

SEMESTER – 6		
Name of the Paper : **CC13**		
Name of the Course : Probability, Statistics		
Course Code : HMAT6CC13N		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course a student will acquire knowledge

- on various aspects of the theory of Probability, as a continuation of their previous concepts from plus two level .
- and skill for solving problems related to various probability distribution functions.
- and skill for solving problems on certain mathematical topics of Statistics.

Detailed Syllabus

Group – A [35 Marks]

(Probability)

1. Random experiment, Sample space, probability as a set function, probability axioms, probability space. Finite sample spaces. Conditional probability, Bayes theorem, independence. Real random variables (discrete and continuous), cumulative distribution function, probability mass/density functions, mathematical expectation, moments, moment generating function, characteristic function. Discrete distributions: Binomial, Poisson, geometric, negative binomial, Continuous distributions: uniform, normal, exponential, beta and gamma function.

2. Joint cumulative distribution function and its properties, joint probability density functions, marginal and conditional distributions, expectation of function of two random variables, moments, covariance, correlation coefficient, independent random variables, joint moment generating function (jmgf) and calculation of covariance from jmgf, characteristic function. Conditional expectations, linear regression for two variables, regression curves. Bivariate normal distribution.

3. Chebyshev's inequality, Convergence in Probability, statement and interpretation of weak law of large numbers and strong law of large numbers. Central limit theorem for independent and identically distributed random variables with finite variance.

Group – B [30 Marks]
(Statistics)

1. **Sampling and Sampling Distributions**: Populations and Samples, Random Sample, distribution of the sample, simple random sampling with and without replacement. Sample characteristics. Statistic, Sample moments. Sample variance, Sampling from the normal distributions, Chi-square, t and F-distributions, sampling distribution of
$$\bar{X},\ s^2,\ \frac{\sqrt{n}}{s}(\bar{X}-\mu),\ \frac{\sqrt{n}}{\sigma}(\bar{X}-\mu).$$

2. **Estimation of parameters**: Point estimation. Interval Estimation- Confidence Intervals for mean and variance of Normal Population. Mean-squared error. Properties of good estimators - unbiasedness, consistency, sufficiency, Minimum-Variance Unbiased Estimator (MVUE).

3. **Method of Maximum likelihood**: Likelihood function, ML estimators for discrete and continuous models.

4. **Statistical hypothesis**: Simple and composite hypotheses, null hypotheses, alternative hypotheses, one sided and two-sided hypotheses. The critical region and test statistic, type I error and type II error, level of significance. Power function of a test, most powerful test. Simple hypothesis versus simple alternative: Neyman-Pearson lemma (Statement only).

5. **Bivariate frequency Distribution**: Bivariate data, Scatter diagram, Correlation, Linear Regression, principle of least squares and fitting of polynomials and exponential curves.

References:

1. A Gupta – Ground work of Mathematical Probability and Statistics, Academic publishers, Kolkata
2. Alexander M Mood, Franklin A Graybill and Duane C Boes – Introduction to the Theory of Statistics, Tata McGraw Hill (3rd Edition, 2007).
3. E Rukmangadachari – Probability and Statistics, Pearson (2012)
4. A Banerjee, S K De and S Sen – Mathematical Probability, U N Dhur & Sons Private Ltd, Kolkata
5. S K De and S Sen – Mathematical Statistics, U N Dhur & Sons Private Ltd, Kolkata

Question Pattern for Semester – 6 Examinations

(Course Code: HMAT6CC13N & Paper: CC13)

Group – A (Probability, 35 marks)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

(ii) Answer **five** questions of 5 marks each from 7 given questions. Each may contain further parts.

Group – B (Statistics, 30 marks)

(i) Answer **three** objective / MCQ type questions of 2 marks each from 4 given questions.

(ii) Answer **four** questions of 6 marks each from 6 given questions. Each question may contain further parts.

SEMESTER – 6		
Name of the Paper : **CC14**		
Name of the Course : Numerical Methods (Theory), Numerical Methods (Laboratory)		
Course Code : HMAT6CC14L		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 50+20*	
Practical	Credits : 1 Full Marks : 30	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course a student will acquire

- fundamental theoretical knowledge on various aspects of the theory of numerical analysis, that will lay the foundation for solving such problems via computer programming to be done side by side using their knowledge of programming language C already acquired from the course DSE01 of Semester – 5.
- basic skill for solving problems (both on paper and via computer) related to various numerical methods on interpolation, numerical differentiation and integration, differential equations and finding roots of an equation.
- basic knowledge and computer oriented skill for solving problems related to certain topics of numerical linear algebra.

Detailed Syllabus

Numerical Methods (Theory) [50 Marks]

1. Representation of real numbers – floating point and fixed point. Sources of Errors, Rounding of numbers, significant digits and Error Propagation in arithmetic operations. Numerical Algorithms - stability and convergence.
2. Approximation: Classes of approximating functions, Types of approximations - polynomial approximation, The Weierstrass polynomial approximation theorem (statement only).
3. Interpolation: Lagrange and Newton's methods. Error bounds. Finite difference operators. Newton (Gregory) forward and backward difference interpolation.
4. Central Interpolation: Gauss forward and backward interpolation formulae, Stirling's and Bessel's formulas. Different interpolation zones, Error estimation. Hermite interpolation.
5. Numerical differentiation: Methods based on interpolations, methods based on finite differences.
6. Numerical Integration: Newton–Cotes formula, Trapezoidal rule, Simpson's $\frac{1}{3}$-rd rule, Simpson's $\frac{3}{8}$-th rule, Weddle's rule, Boole's Rule, midpoint rule. Composite trapezoidal rule, composite Simpson's $\frac{1}{3}$-rd rule, composite Weddle's rule. Gaussian quadrature formula.

7. Transcendental and polynomial equations: Bisection method, Secant method, method of Regula-Falsi, fixed point iteration, Newton-Raphson method. Condition of convergence (if any), Order of convergence, Rate of convergence of these methods. Modified Newton-Raphson method for multiple roots, Complex roots of an algebraic equation by Newton-Raphson method.
8. System of linear algebraic equations: Direct methods: Gaussian elimination and Gauss Jordan methods, Pivoting strategies. Iterative methods: Gauss Jacobi method, Gauss Seidel method and their convergence analysis. Matrix inversion: Gaussian elimination (operational counts).
9. The algebraic Eigen value problem: Power method.
10. Ordinary differential equations: The method of successive approximations (Picard), Euler's method, the modified Euler method, Runge-Kutta methods of orders two and four.

References:

1. K E Atkinson – Elementary Numerical Analysis, John Wiley & Sons (1985)
2. F B Hildebrand – Introduction to Numerical Analysis, Dover Publications, INC (2nd Edition, 1974)
3. D C Sanyal and K Das – A Text Book Of Numerical Analysis, U N Dhur & Sons Private Ltd, Kolkata
4. Michelle Schatzman – Numerical Analysis – A Mathematical Introduction, Oxford University Press (2002)
5. M K Jain, S R K Iyengar and R K Jain – Numerical Methods for Scientific and Engineering Computation, New Age International (P) Ltd., New Delhi (1996)

Question Pattern for Semester – 6 Examinations

(Course Code: HMAT6CC14L [Theory] & Paper: CC14)

(i) Answer **four** objective / MCQ type questions of 2 marks each from 5 given questions.

(ii) Answer **six** questions of 7 marks each from 8 given questions. Each question may contain further parts.

Detailed Syllabus

Numerical Methods (Laboratory) [30 Marks]

List of practical (using C)

1. **Interpolation**
   (i) Lagrange Interpolation
   (ii) Newton's forward, backward and divided difference interpolations
2. **Numerical Integration**
   (i) Trapezoidal Rule
   (ii) Simpson's one third rule
3. **Solution of transcendental and algebraic equations**
   (i) Bisection method
   (ii) Newton Raphson method (Simple root, multiple roots, complex roots)
   (iii) Method of Regula-Falsi.
4. **Solution of system of linear equations**
   (i) Gaussian elimination method
   (ii) Matrix inversion method
   (iii) Gauss-Seidel method
5. **Method of finding Eigenvalue by Power method** (up to 4×4)
6. **Fitting a Polynomial Function** (up to third degree)
7. **Solution of ordinary differential equations**
   (i) Euler method
   (ii) Modified Euler method
   (iii) Runge-Kutta method (order 4)

Question Pattern for Semester – 6 Examinations

(Course Code: HMAT6CC14L [Practical] & Paper: CC14)

(i) Answer **four** questions of 5 marks each from 7 given questions. All problems to be done on computer by using C programming only. Allotted time is three hours.

(ii) 5 marks reserved for practical note book.

(iii) 5 marks reserved for viva voce.

Discipline Specific Electives (DSE)

SEMESTER – 5	
Name of the Paper : **DSE01**	
Name of the Course : Computer Programming with C & Scientific Computing with R	
Course Code : HMAT5DS11L	
Full Marks : 100	Credit: 6
Theory	Credits : 4 Full Marks : 50+20*
Number of theory classes required : 60	
Practical	Credits : 2 Full Marks : 30
Number of practical classes required : 40	
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance	

Course Outcomes: At the end of studying this course a student

- will acquire fundamental operational knowledge on various aspects of the important computer programming language C and side by side they will be exposed to hands on computer oriented skill development using the language taught.
- will acquire basic skill for solving problems using C language in the computer laboratory, that will help them to solve such problems on various Numerical Methods later in CC14 of Semester – 6.
- will learn the fundamental commands and structure of R & C. The course covers the basic syntax and semantics of R & C, including basic data types, variables, control structures and functions or similar concepts, and visualization of results and processed data.

Detailed Syllabus

Computer Programming with C & Scientific computing with R
[Theory, 50 Marks]

Group – A [25 Marks]
(Programming with C)

1. An overview of theoretical computers, history of computers, overview of architecture of computer, compiler, assembler, machine language, high level language, object oriented language, programming language and importance of C programming.
2. Constants, Variables and Data type of C-Program: Character set. Constants and variables data types, expression, assignment statements, declaration.
3. Operation and Expressions: Arithmetic operators, relational operators, logical operators.
4. Decision Making and Branching: decision making with if statement, if-else statement, Nesting if statement, switch statement, break and continue statement.

5. Control Statements While statement, do-while statement, for statement.
6. Arrays: One-dimension, two-dimension and multidimensional arrays, declaration of arrays, initialization of one and multi-dimensional arrays.
7. User-defined Functions: Definition of functions, Scope of variables, return values and their types, function declaration, function call by value, nesting of functions, passing of arrays to functions, Recurrence of function.
8. Introduction to Library functions: $stdio.h$, $math.h$, $string.h$, $stdlib.h$, $time.h$ etc.

Group – B [25 Marks]

(Scientific computing with *R*)

1. Introduction to R, Installation Procedure, Use of R as a Calculator, Numerical and symbolic computations using mathematical functions such as square root, trigonometric functions, logarithms, exponentiations etc.
2. Graphical representations of few functions through plotting in a given interval, like plotting of polynomial functions, trigonometric functions, Plots of functions with asymptotes, superimposing multiple graphs in one plot like plotting a curve along with a tangent on that curve (if it exists), polar plotting of curves.
3. R Commands for differentiation, higher order derivatives, plotting $f(x)$ and together, integrals, definite integrals etc.
4. Introduction to Programming in R, relational and logical operators, conditional statements, loops and nested loops, without using inbuilt functions write programs for average of integers, mean, median, mode, factorial, checking primes, checking next primes, finding all primes in an interval, finding gcd, lcm, finding convergence of a given sequence, etc.
5. Use of inbuilt functions that deal with matrices, determinant, inverse of a given real square matrix (if it exists), solving a system of linear equations, finding roots of a given polynomial, solving differential equations.

References:

1. C Xavier – C Language and Numerical Methods, New Age International Pvt. Ltd., New Delhi (2003)
2. C B Gottfried – Programming with C, Schaum's outlines (2nd Edition, 1996)
3. E Balagurusamy – Programing in Ansi C, Tata McGraw-Hill Education (2004)
4. Biswadip Pal – C Programming Language, Techno World (2019)
5. P. Dalgaard – Introductory Statistics with R, Springer (2nd Edition, 2008)
6. J. Maindonald & J. Braun – Data Analysis and Graphics Using R: An Example-based Approach, Cambridge University Press (3rd Edition , 2010)

<u>Question Pattern for Semester – 5 Examinations</u>

(Course Code: HMAT5DSE11L & Paper: DSE01)

(Theory)

Group – A (Programming with C, 25 marks)

(i) Answer two objective / MCQ type questions of 2 marks each from 3 given questions.

(ii) Answer three questions of 7 marks each from 5 given questions. Each question may contain further parts.

Group – B (Scientific computing with R, 25 marks)

(i) Answer two objective / MCQ type questions of 2 marks each from 3 given questions.

(ii) Answer three questions of 7 marks each from 5 given questions. Each question may contain further parts.

Computer Programming with C & Scientific computing with R (Laboratory)

[Practical, 30 Marks]

- Standard mathematical problem solving using C and R programming.
- Some hands on examples should be included.

Question Pattern for Semester – 5 Examinations

(Course Code: HMAT5DSE11L & Paper: DSE01)
(Practical)

(i) Answer **four** questions of 5 marks each from 7 given questions. All problems to be done on computer by using C and R programming only. Allotted time is three hours.

(ii) 5 marks reserved for practical note book.

(iii) 5 marks reserved for viva voce.

SEMESTER – 5		
Name of the Paper : **DSE01**		
Name of the Course : Graph Theory and Number Theory		
Course Code : HMAT5DS12N		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this paper a student will acquire

- fundamental knowledge on certain topics of Graph theory which is one of the relatively modern topics of mathematics, having a wide range of applications. This will help the motivated students to pursue this subject at the higher level of study in post graduate.
- basic knowledge and problem solving skills on certain topics of Number Theory as a continuation of what they have already learned in CC01.
- some initial exposure to the upcoming modern branch of Cryptology having a large scope of application in our present day life through the standard protocols of RSA cryptosystem and digital signature. Application of group theory in the making of the check digit of an Aadhar card will help them to appreciate the applicability of the so-called abstract mathematics in our day to day life.

Detailed Syllabus

Group – A (35 Marks)
(Graph Theory)

1. Definition of undirected graphs, Using of graphs to solve different puzzles and problems. Multigraphs. Walks, Trails, Paths, Circuits and cycles, Eulerian circuits and paths. Eulerian graphs, example of Eulerian graphs. Hamiltonian cycles and Hamiltonian graphs.
2. Weighted graphs and Travelling Sales Persons' Problem. Dijkstra's algorithm to find shortest path.
3. Definition of Trees and their elementary properties. Definition and elementary characteristics of Planar graphs, Kuratowski's graphs.
4. Adjacency matrix of a graph, Graphs and eigenvalues, Regular and strongly regular graphs.

Group – B (30 Marks)
(Number Theory)

1. Linear Diophantine Equation.
2. The Arithmetic of $\mathbb{Z}_p$, p a prime, pseudo prime and Carmichael Numbers, Fermat Numbers, Perfect Numbers, Mersenne Numbers.
3. Primitive roots, the group of units $\mathbb{Z}_n$, the existence of primitive roots, applications of primitive roots, the algebraic structure of $\mathbb{Z}_n$.
4. Quadratic residues and non-quadratic residues, Legendre symbol, proof of the law of quadratic reciprocity, Jacobi symbols.
5. Applications: RSA Cryptosystem and digital signature, Dihedral group D_5 and check digit of Aadhar Number.

References

1. N. Deo – Graph Theory with Application to Engineering and Computer Science, Prentice Hall of India, New Delhi, 1990.
2. D.S. Malik and M.K. Sen – Discrete Mathematics: Theory and application
3. F. Harary – Graph Theory; Narosa Publishing House, New Delhi, 2001.
4. Helen Sapiro – Linear Algebra and matrices AMS.
5. Richard A Mollin – Advanced Number Theory with Applications, CRC Press, A Chapman & Hall Book.
6. Gareth A Jones and J Mary Jones – Elementary Number Theory, Springer International Edition.
7. Neal Koblitz – A course in number theory and cryptography, Springer-Verlag, 2nd edition.
8. D. M. Burton – Elementary Number Theory, Wm. C. Brown Publishers, Dulreque, Lowa, 1989.
9. Kenneth. H. Rosen – Elementary Number Theory & Its Applications, AT&T Bell Laboratories, Addition-Wesley Publishing Company, 3rd Edition.
10. Kenneth Ireland & Michael Rosen – A Classical Introduction to Modern Number Theory, 2nd edition, Springer-Verlag.
11. M.K. Sen, Shamik Ghosh, Parthasarathi Mukhopadhyay, S. K. Maity – Topics in Abstract Algebra, (Revised 3rd Edition), Universities Press, 2020

Question Pattern for Semester – 5 Examinations

(Course Code: HMAT5DS12N & Paper: DSE01)

Group A (Graph Theory, 35 Marks):

(i) Answer **five** questions of objective / MCQ type from seven given questions each carrying 2 marks.

(ii) Answer **five** questions from seven given questions each carrying 5 marks. Each question may have further parts.

Group B (Number Theory, 30 Marks):

(i) Answer **five** questions of objective / MCQ type from seven given questions each carrying 2 marks.

(ii) Answer **four** questions from six given questions each carrying 5 marks. Each question may have further parts.

SEMESTER – 5		
Name of the Paper : **DSE02**		
Name of the Course : Advanced Algebra		
Course Code : HMAT5DS21N		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Advanced Algebra [65 Marks]

Course Outcomes: At the end of studying this course a student will acquire

- fundamental knowledge on certain advanced topics of Group and Ring theory as a continuation of what they have already learned in various compulsory core courses. This will help the motivated students to pursue Algebra at the higher level of study in post graduate.
- basic knowledge and problem solving skills on certain advanced topics of linear algebra like the SVD, generalized inverse, Moore-Penrose inverse, Jordan and rational canonical form of a non-diagonalizable matrix etc.
- some knowledge that will help the students to solve problems in various national entrance examinations for their next level of study and also for the NET examination conducted by CSIR after their completion of master's degree.

Detailed Syllabus

Group – A [15 Marks]
(Group Theory)

1. **Group Theory**: Cauchy's theorem for finite Abelian group, fundamental theorem of finite Abelian groups, Group actions, applications of group actions, Generalized Cayley's theorem, Index theorem. Groups acting on themselves by conjugation, Class equation and consequences, p-groups, statement of Sylow's theorems and consequences, Cauchy's theorem, Simple group, simplicity of A_n for $n \geq 5$, non-simplicity tests.

Group – B [20 Marks]
(Ring Theory)

1. **Ring Theory**: Principal ideal domain, principal ideal ring, prime element, irreducible element, greatest common divisor (gcd), least common multiple (lcm), expression of

gcd, examples of a ring R and a pair of elements $a, b \in R$ such that $gcd(a, b)$ does not exist, Euclidean domain, relation between Euclidean domain and principal ideal domain.

2. Ring embedding and quotient field, regular rings and their examples, properties of regular ring, ideals in regular rings.
3. Polynomial rings, division algorithm and consequences, factorization domain, unique factorization domain, irreducible and prime elements in a unique factorization domain, relation between principal ideal domain, unique factorization domain, factorization domain and integral domain, Eisenstein criterion and unique factorization in $Z[x]$.

Group – C [30 Marks]
(Linear Algebra)

1. Affine combination, affine subspace or Flat, Quotient space of a finite dimensional vector space, quotient transformation, dimension theorem.
2. Function space and Fourier series, Fourier Coefficients in the light of Linear Algebra
3. Positive definite and semi-definite operators, positive (definite) matrices, square root of a positive (definite) matrix, Cholesky Decomposition.
4. Invariant factors and minimal polynomial of a square matrix, Smith Normal form, Companion matrix, Rational Canonical form (computation only), Elementary divisors of a square matrix, Jordan Blocks, Generalized eigenvector, Jordan canonical form (computation only), (*Structure theorems for existence via Module theory is NOT covered in the syllabus*) , Conversion from one form to another.
5. Singular values of a $m \times n$ matrix, Singular Value Decomposition (SVD) of (i) a linear Transformation, (ii) square non-singular matrix, (iii) square singular matrices, (iv) any general matrix, reduced SVD, application in Image processing, condition number of an invertible matrix.
6. Polar Decomposition of a matrix directly and via SVD, Polar decomposition of a linear operator (independent of SVD).
7. Generalized inverses of a rectangular matrix, g-inverse, reflexive g-inverse (pseudoinverse), minimum norm g-inverse, least square g-inverse, Moore-Penrose (MP) inverse, Properties of MP inverse, best approximation in terms of MP inverse. Finding the best solution to $Ax = b$ when it is not solvable usually, minimum norm least square solution of $Ax = b$ and MP inverse of linear Transformation.

References:

1. M K Sen, S Ghosh, P Mukhopadhyay and S Maity – Topics in Abstract Algebra, University Press, Hyderabad (Revised 3rd Edition, 2020)
2. D S Malik, John M Mordeson and M K Sen – Fundamentals of Abstract Algebra, McGraw Hill, New York (1997)
3. John B Fraleigh – A First Course in Abstract Algebra, Pearson (7th Edition, 2002)
4. M Artin – Abstract Algebra, Pearson (2nd Edition, 2011)

5. Joseph A Gallian – Contemporary Abstract Algebra, Narosa Publishing House, New Delhi (4th Edition, 1999)
6. Joseph J Rotman – An Introduction to the Theory of Groups, Springer (4th Edition, 1995)
7. I N Herstein – Topics in Algebra, Wiley Eastern Limited, India (1975)
8. Kenneth Hoffman and Ray Alden Kunze – Linear Algebra, Prentice-Hall of India Pvt. Ltd. (2nd Edition, 1971)
9. Stephen H Friedberg, Arnold J Insel and Lawrence E Spence – Linear Algebra, Prentice- Hall of India Pvt. Ltd., New Delhi (4th Edition, 2004)
10. Gilbert Strang – Linear Algebra and its Applications, Thomson (2007)
11. Gilbert Strang – Fundamentals of Linear Algebra, Cambridge University Press (2007)
12. S Lang – Introduction to Linear Algebra, Springer (2nd Edition, 2005)
13. S Kumaresan – Linear Algebra- A Geometric Approach, Prentice Hall of India (1999)
14. R.B. Bapat – Linear Algebra and Linear Models (Third Edition), HBA, (2012)
15. Vivek Sahai, Vikas Bist – Linear Algebra (Second Edition), Narosa, (2013)
16. Harry Dym – Linear Algebra in Action,(Indian Edition) American Mathematical Society (2014)
17. Helen Sapiro – Linear Algebra and Matrices, Topics for a Second Course, (Indian Edition) American Mathematical Society (2015)
18. Mark Gockenbach-Finite Dimensional Linear Algebra, CRC press, (2010)
19. Lawrence Spence, Arnold Insel, Stephen Friedberg - Elementary Linear Algebra, A Matrix Approach, (Second Edition, Indian Subcontinent Reprint), Pearson (2019)
20. Karim Abadir, Jan Magnus – Matrix Algebra, Cambridge (2005)

Question Pattern for Semester – 5 Examinations

(Course Code: HMAT5DS21N & Paper: DSE02)

Group – A (Group Theory, 15 marks)

(i) Answer **one** question of 8 marks from 2 given questions.

(ii) Answer **one** question of 7 marks from 2 given questions. Each question may contain further parts including objective and MCQ type.

Group – B (Ring Theory, 20 marks)

Answer **two** questions of 10 marks each from 4 given questions. Each question may contain further parts including objective and MCQ type.

Group – C (Linear Algebra, 30 marks)

Answer **three** questions of 10 marks each from 5 given questions. Each question may contain further parts including objective and MCQ type.

SEMESTER – 5	
Name of the Paper : **DSE02**	
Name of the Course : Advanced Mechanics	
Course Code : HMAT5DS22N	
Full Marks : 100	Credit: 6
Theory	Credits : 5 Full Marks : 65+20*
Tutorial	Credits : 1 Full Marks : 15
Number of classes required : 100	
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance	

Course Outcomes: At the end of studying this course a student will acquire

- fundamental knowledge on Generalized coordinates and its applications.
- knowledge on non-holonomic systems, Hamiltonian etc. which will help the students in the next level of study in Applied Mathematics.
- some knowledge on generating function, Poisson Bracket, Hamilton's characteristics function etc.

Detailed Syllabus

Advanced Mechanics [65 Marks]

1. Degrees of freedom, reactions due to constraints. D' Alembert's principle, Lagrange's first kind equations, Generalized coordinates, Generalized forces, Lagrangian, Second kind Lagrange's equations of motion. Cyclic coordinates, velocity dependent potential. Principle of energy and Rayleigh's dissipation function.
2. Action Integral, Hamilton's principle, Lagrange's equations by variational methods. Hamilton's principle for non-holonomic system. Symmetry properties and conservation laws. Noether's theorem. Canonically conjugate coordinates and momenta. Legendre transformation, Routhian approach and Hamiltonian.
3. Hamilton's equations from variational principle. Poincare-Cartan integral invariant. Principle of stationary action and Fermat's principle.
4. Canonical transformation, Generating function, Poisson Bracket. Equations of motion, Action-angle variables. Hamilton-Jacobi's equation, Hamilton's principal function, Hamilton's characteristics function and Liouville's theorem.

References:

1. H Goldstein – Classical Mechanics, Narosa Publ., New Delhi.
2. N C Rana and P S Joag – Classical Mechanics, Tata McGraw Hill.
3. E T Whittaker – A Treatise of Analytical Dynamics of Particles and Rigid Bodies, Cambridge Univ. Press, Cambridge.

4. T W B Kibble and F H Berkshire – Classical Mechanics, Addison-Wesley Longman.
5. V I Arnold – Mathematical Methods of Classical Mechanics, Springer.
6. N G Chetaev –Theoretical Mechanics, Springer.
7. M Calkin – Lagrangian and Hamiltonian Mechanics, World Sci. Publ., Singapore.
8. J L Synge and B A Griffith – Principles of Mechanics, McGraw Hill.

Question Pattern for Semester – 5 Examinations

(Course Code: HMAT5DS22N & Paper: DSE02)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

(ii) Answer **one** question of 10 marks each from 2 given questions from Article 1. Each question may contain further parts.

(iii) Answer **three** questions of 10 marks each from 5 given questions from Articles 2 and 3. Each question may contain further parts.

(iv) Answer **three** questions of 5 marks each from 5 given questions from Article 4. Each question may contain further parts.

SEMESTER – 6	
Name of the Paper : **DSE03**	
Name of the Course : Linear Programming Problem & Game Theory	
Course Code : HMAT6DS31N	
Full Marks : 100	Credit: 6
Theory	Credits : 5 Full Marks : 65+20*
Tutorial	Credits : 1 Full Marks : 15
Number of classes required : 100	
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance	

Course Outcomes: At the end of studying this paper a student will acquire

- fundamental knowledge on the theory of basic and basic feasible solutions and their properties, convex sets based on the knowledge of linear algebra studied in previous semesters.
- the skills on the solution of a Linear Programming Problem by Simplex Method. Also acquire knowledge on duality, transportation problem, assignment problem and travelling salesman problem.
- some knowledge on the basic theory of game problems and their solution by different methods which has many applications in Economics.

Detailed Syllabus

Group – A (50 Marks)

(Linear Programming Problem)

1. **Mathematical Preliminaries:** Basic solution and Basic Feasible Solution (B.F.S) with reference to Linear Programming Problem (L.P.P.). Matrix formulation of a L.P.P. Degenerate and Non-degenerate B.F.S.
2. **Convex Sets and related theorems:** Hyperplane, convex set, extreme points, convex hull and convex polyhedron. Supporting and separating hyperplanes. The collection of feasible solutions of a L.P.P. constitute a convex set. Every extreme point of the convex set of feasible solutions correspond to a B.F.S. and conversely. The objective function has its optimal value at an extreme point of the convex polyhedron generated by the set of feasible solutions (the convex polyhedron may also be unbounded). In the absence of degeneracy, if a L.P.P. admits of an optimal solution, then at least one B.F.S. must be optimal. Reduction of a F.S. to a B.F.S.
3. **Simplex Algorithm:** Slack and surplus variables. Standard form of a L.P.P. Theory of simplex method. Feasibility and optimality conditions. Charnes' Big-M Method. Two phase simplex method. Application of Phase-I simples method in finding the inverse of

a non-singular matrix and solution of a system of equations. Degeneracy in L.P.P. and its resolution – Charnes' perturbation method and generalized simplex method.

4. **Duality in Linear Programming:** Primal-Dual problems. Functional properties of duality – The dual of the dual is the primal. Duality Theorems. Relation between the objective values of the dual and the primal problems. Relation between their optimal values. Complementary slackness. Duality and Simplex method – Their applications.
5. **Transportation problem:** Mathematical formulation, existence of a feasible solution and an optimal solution of a Transportation Problem (T.P). Determination of an initial Basic Feasible Solution of a T.P. Optimality Test – Modified Distribution Method (MODI method). Degeneracy in a T.P. Unbalanced T.P. Maximization in a T.P. prohibited routes in a T.P.
6. **Assignment Problem:** Mathematical formulation, Hungarian method of solving an Assignment Problem (A.P). Unbalanced and Maximization T.P. Prohibited assignments.
7. **Travelling Salesman problem**.

Group – B (15 Marks)

(Game Theory)

1. Concept of a game problem. Rectangular games. Pure strategy and mixed strategy. Two-person zero-sum game problems. Maximin-Minimax Principle. Saddle point and its existence – related theorems.
2. Game without saddle point – related theorems. Algebraic method. Graphical solution, Principle of dominance. Optimal strategies and value of the game.
3. Reduction of a game problem to a L.P.P. Fundamental Theorem of Rectangular Game Problem and its applications.

References:

1. G Hadley – Linear Programming, Addison-Wesley Publishing Company, London (1972)
2. Hamdy A Taha – Operations Research – An Introduction, Prentice hall of India Pvt. Ltd., New Delhi (1999)
3. N S Kambo – Mathematical Programming Techniques, Affiliated East-West Press Pvt. Ltd., New Delhi (1997)
4. P K Gupta and Manmohan – Linear Programming and Theory of Games, Sultan Chand & Sons, New Delhi (1997)
5. N Mandal & B Pal – Linear Programming and Game Theory, Techno World, Kolkata (2021).
6. J G Chakravorty & P R Ghosh – Linear Programming & Game Theory, Moulik Library Publisher, Kolkata
7. S Mukhopadhyay – Linear Programming with Game Theory, Academic Publishers, Kolkata.

<u>Question Pattern for Semester – 6 Examinations</u>

(Course Code: HMAT6DS31N & Paper: DSE03)

Group – A (Linear Programming Problem, 50 marks)

(i) Answer **five** objective / MCQ type questions of 2 marks each from 6 given questions.

(ii) Answer **five** questions of 8 marks each from 7 given questions. Each question may contain further parts.

Group – B (Game Theory, 15 marks)

Answer **three** questions of 5 marks each from 5 given questions.

SEMESTER – 6		
Name of the Paper : DSE03		
Name of the Course : Advanced Analysis		
Course Code : HMAT6DS32N		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Advanced Analysis

Course Outcomes: At the end of studying this paper a student will acquire

- fundamental knowledge on certain advanced topics of Real analysis which will serve as a bridge course with the subject of Functional Analysis in future. This will help the motivated students to pursue this subject at the higher level of study in post graduate.
- some initial exposure to Hilbert Space.
- some initial exposure to some of the advanced topics of Complex Analysis as a continuation of the basic course of Complex analysis that they are already exposed to through one of the core courses. This will help the motivated students to pursue this subject at the higher level of study in post graduate.

Detailed Syllabus

Group – A (40 Marks)
(Real Analysis)

1. **Cardinal Number**: Concept of Cardinal number of an infinite set, order relation of Cardinal numbers, Schröeder-Bernstein theorem, the set 2^A, Axiom of choice, arithmetic of Cardinal numbers, Cardinality of Cantor set, continuum hypothesis.
2. **Absolutely continuous functions and functions of bounded variation**: Absolutely continuous functions, properties of absolutely continuous functions, Lipschitz condition and absolutely continuous function. Definition of bounded variation, examples and properties of functions of bounded variation, Relation between absolute continuity and bounded variation.
3. **Summability**: Introduction, Cesaro Summability of order one and two.
4. **Introduction to Banach Space**: Normed linear space, completeness of a Normed linear space, Definition of a Banach space with examples, quotient space.
5. **Introduction to Hilbert Space**: Inner product, Definition and examples of a Hilbert space.

Group – B (25 Marks)

(Complex Analysis)

1. Proof of Cauchy – Goursat Theorem and its consequences, Cauchy Integral formula for derivatives, Statement of Morera's Theorem, and Liouville's Theorem with the application.
2. Zero of Analytic functions, various kinds of singularities of complex-valued function, removable singularity, pole, essential singularity, classification of singularities using the Laurant series, Statement of Residue Theorem and its application to evaluation of certain definite integrals.

References:

1. J. B Randolph, Basic Real and Abstract Analysis, Academic Press
2. Tom M. Apostol, Mathematical Analysis, Addison Wesley
3. Elias M Stein and Rami Shakarchi, Real Analysis – Measure Theory, Integration, and Hilbert Spaces, PRINCETON UNIVERSITY PRESS
4. H.L. Royden, P.M. Fitzpatrick, Real Analysis, PHI
5. Waclaw Sierpinski, Cardinal and Ordinal Numbers, Warszawa
6. William O Ray, Real Analysis, Prentice-Hall
7. R. V. Churchill and J. W. Brown – Complex Variables and Applications, McGraw-Hill, New York.
8. S Kumaresan – A Pathway to Complex Analysis, Techno World.
9. S Ponnusamy – Foundations of Complex Analysis, Springer.
10. S. Ponnusamy and Herb Silverman – Complex Variable with Applications, Birkhäuser.
11. J. B. Conway – Functions of One Complex Variable, Narosa Publishing, New Delhi.

Question Pattern for Semester – 6 Examinations

(Course Code: HMAT6DS32 & Paper: DSE03)

GROUP – A (Real Analysis, 40 Marks)

(i) Answer **four** objective / MCQ type questions of 2 marks each from five given questions.

(ii) Answer **four** questions of 3 marks each from five given questions.

(iii) Answer **four** questions of 5 marks from six given questions. Each question may contain further parts.

GROUP – B (Complex Analysis, 25 Marks)

(i) Answer **three** objective / MCQ type questions of 2 marks each from four given questions.

(ii) Answer **three** questions of 3 marks each from four given questions.

(iii) Answer **two** questions of 5 marks from three given questions. Each question may contain further parts.

SEMESTER – 6		
Name of the Paper : **DSE04**		
Name of the Course : Mathematical Logic		
Course Code : HMAT6DS41N		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this paper a student

- will be able to understand concepts of mathematical logic for analysing propositions and proving theorems.
- will acquire basic knowledge on Boolean algebra and applications.
- will acquire fundamental knowledge on Propositional Logic and a connection with Boolean algebra.
- will acquire fundamental knowledge on Predicate Logic.

Detailed Syllabus

Mathematical Logic [65 Marks]

1. **Introduction:** Boolean algebra – Definition (Huntington′s postulates), examples, Duality principle, Theorems on Boolean algebra, two element Boolean algebra. Boolean function.
2. **General Notions**: What is Logic? Why is it called Mathematical? A brief history of Logic. Formal language, object and meta language, general definition of a Formal Theory/Formal Logic.
3. **Propositional Logic** : Truth tables, tautology, adequate set of connectives, Disjunctive Normal Form, Conjunctive Normal Form, applications to switching circuits, Formal theory for propositional calculus, derivation, proof, theorem, deduction theorem, semantics, logical consequence, consistency, maximal consistency, Leindenbaum lemma, soundness and completeness theorems, Lindenbaum construction of Logic of Propositional Logic.
4. **Predicate Logic** : First order language, symbolization of natural language sentences by the first order language, free and bound variables, Interpretation, satisfiability, Truth, Validity, models, Axiomatic system of Predicate Logic, theorems and derivations, deduction theorem, choice rule, equivalence theorem, replacement theorem, Prenex Normal form (Definition and examples), Soundness theorem, Completeness theorem (statement only), Compactness theorem (statement only).

References:

1. Elliott Mendelson – Introduction to mathematical logic, Chapman & Hall, London.
2. Angelo Margaris – First order mathematical logic, Dover publications Inc, New York.
3. S C Kleene – Introduction to Metamathematics, Amsterdam, Elsevier.
4. J H Gallier – Logic for Computer Science, John Wiley & Sons.
5. H B Enderton – A mathematical introduction to logic, Academic Press, New York.

Question Pattern for Semester – 6 Examinations

(Course Code: HMAT6DS41N & Paper: DSE04)

(i) Answer **seven** objective / MCQ type questions each of 2 marks from eight given such questions.

(ii) Answer **three** questions of 5 marks each from 4 given questions from Articles 1 and 2. Each question may contain further parts.

(iii) Answer **three** questions of 7 marks each from 4 given questions from Article 3. Each question may contain further parts.

(iv) Answer **three** questions of 5 marks each from 4 given questions from Article 4. Each question may contain further parts.

SEMESTER – 6		
Name of the Paper : **DSE04**		
Name of the Course : Differential Geometry		
Course Code : HMAT6DS42N		
Full Marks : 100		Credit: 6
Theory	Credits : 5 Full Marks : 65+20*	
Tutorial	Credits : 1 Full Marks : 15	
Number of classes required : 100		
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance		

Course Outcomes: At the end of studying this course, students will acquire

- a fairly good knowledge of tensor analysis useful for the study of differential geometry.
- knowledge of the theory of space curves and surfaces.
- knowledge of the fundamental equations of surface theory and geodesics and their properties, which may motivate them to pursue further studies in the subject of differentiable manifolds having application in different areas of Mathematics and Physics.

Detailed Syllabus

Differential Geometry (65 Marks)

1. Tensor Analysis: Different transformation laws, Properties of tensors, Metric tensor, Riemannian space, Covariant Differentiation, Einstein space.
2. Theory of space curves: Space curves. Planar curves, curvature, torsion and Serret-Frenet formula. Osculating circles and spheres. Existence of space curves. Evolutes and involutes of curves.
   Theory of surfaces: Parametric curves on surfaces. Direction coefficients. First and second Fundamental forms. Principal and Gaussian curvatures. Lines of curvature, Euler's theorem. Rodrigue's formula. Conjugate and asymptotic lines.
3. Developables: Developables associated with space curves and curves on surfaces. Minimal surfaces. Geodesics. Canonical geodesic equations. Nature of geodesics on a surface of revolution. Clairaut's theorem. Normal property of geodesics. Torsion of a geodesic. Geodesic curvature. Gauss-Bonnet theorem.

References:

1. T. J. Willmore – An Introduction to Differential Geometry, Dover Publications.
2. B. O'Neill – Elementary Differential Geometry, 2nd Ed, Academic Press.
3. C. E. Weatherburn – Differential Geometry of Three Dimensions, Cambridge University Press.
4. D. J. Struik – Lectures on Classical Differential Geometry, Dover Publications.
5. S. Lang – Fundamentals of Differential Geometry, Springer.
6. B. Spain – Tensor Calculus: A Concise Course, Dover Publications.

7. L. P. Eisenhart – An Introduction to Differential Geometry (with the use of Tensor Calculus), Princeton University Press.
8. I. S. Sokolnikoff – Tensor Analysis, Theory and Applications to Geometry and Mechanics of Continua, 2nd Edition, John Wiley and Sons.

Question Pattern for Semester – 6 Examinations

(Course Code: HMAT6DS42N & Paper: DSE04)

(i) Answer **five** objective / MCQ type questions of 2 marks each from six given questions

(ii) Answer **two** questions of 5 marks each from three given questions from Article 1. Each question may have further parts.

(iii) Answer **three** questions of 8 marks each from four given questions from Article 2. Each question may have further parts.

(iv) Answer **three** questions of 7 marks each from four given questions from Article 3. Each question may have further parts.

Generic Elective (GE)

[To be taken by the students of other discipline]

Course Structure: Semester-wise distribution of Courses

Semester	Course Name	Course Code	Credits
1	Modern Algebra, Differential Calculus, Linear Programming	HMAT1GE01N	6
2	Modern Algebra, Differential Calculus, Linear Programming	HMAT2GE01N	6
3	Linear Algebra, Integral Calculus, Differential Equations, Laplace Transforms	HMAT3GE02N	6
4	Linear Algebra, Integral Calculus, Differential Equations, Laplace Transforms	HMAT4GE02N	6
	Grand Total		24

Semester-wise detailed syllabus

SEMESTER – 1	
Name of the Paper: **GE01**	
Name of the Course : Modern Algebra, Differential Calculus, Linear Programming	
Course Code : HMAT1GE01N	
Full Marks : 100	Credit: 6
Theory	Credits : 5 Full Marks : 65+20*
Tutorial	Credits : 1 Full Marks : 15
Number of classes required : 100	
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance	

&

SEMESTER – 2	
Name of the Paper: **GE01**	
Name of the Course : Modern Algebra, Differential Calculus, Linear Programming	
Course Code : HMAT2GE01N	
Full Marks : 100	Credit: 6
Theory	Credits : 5 Full Marks : 65+20*
Tutorial	Credits : 1 Full Marks : 15
Number of classes required : 100	
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance	

Course Outcomes: At the end of studying this course a student will acquire

- fundamental knowledge on the elementary theory of Group, Cyclic Group, Ring and Field which are necessary for proper understanding of certain topics of various core courses of other major subjects.
- very important knowledge on differential calculus that will help the students to understand the applications of calculus to their respective core courses.
- fundamental knowledge on linear programming problems and their applications as a continuation of their earlier knowledge in plus two level.

Detailed Syllabus

Group – A [15 Marks]
(Modern Algebra)

1. Introduction of Group Theory: Definition and examples taken from various branches (example from number system, residue class of integers modulo n, roots of unity, 2×2 real matrices, non-singular real matrices of fixed order, groups of symmetries of an equilateral triangle, a square). Elementary properties using definition of Group. Definition and examples of sub-group (statement of necessary and sufficient condition and its applications). Definition and examples of cyclic group, properties of cyclic group. Order of an element of a finite group, statement and applications of Lagrange's theorem on finite group.
2. Definition and examples of (i) Ring (ii) Field (iii) Sub-ring (iv) Sub-field.
3. Rank of a matrix: Determination of rank either by considering minors or by sweep-out process. Consistency and solution of a system of linear equations with not more than 3 variables by matrix method.
4. Real Quadratic Form involving not more than three variables, its rank, index and signature (problems only).

Group – B [30 marks]
(Differential Calculus)

1. Sequence of real numbers: Definition of bounds of a sequence and monotone sequence. Limit of a sequence. Statements of limit theorems. Concept of convergence and divergence of monotone sequences – applications of the theorems, in particular, definition of e. Statement of Cauchy's general principle of convergence andits application.
2. Real-valued functions defined on an interval, limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval. Acquaintance (on proof) with the important properties of continuous functions on closed intervals. Statement of existence of inverse function of a strictly monotone function and its continuity.
3. Derivative - its geometrical and physical interpretation. Sign of derivative-Monotonic increasing and decreasing functions. Relation between continuity and derivability.
4. Successive derivative – Leibnitz's theorem and its application.
5. Functions of two and three variables: Geometrical representations. Limit and Continuity (definition only) for function of two variables. Partial derivatives. Knowledge and use of chain rule. Exact Differential (emphasis on solving problems only). Functions of two variables - Successive partial Derivatives: Statement of Schwarz's Theorem on Commutative property of mixed derivatives. Euler's Theorem on homogeneous functions of two and three variables.
6. Statement of Rolle's Theorem and its geometrical interpretation. Mean value theorems of Lagrange and Cauchy. Statements of Taylor's and Maclaurin's Theorems with

Lagrange's and Cauchy's from of remainders. Taylor's and Maclaurin's Infinite series of functions like e^x, $sin\ x$, $cos\ x$, $(1 + x)^n$, $\log_e(1 + x)$ with restrictions wherever necessary.

7. Indeterminate Forms: L'Hospital's Rule: Statement and Problems only.
8. Application of the principle of Maxima and Minima for a function of single variable in geometrical, physical and other problems.
9. Maxima and minima of functions of not more than three variables. Lagrange's Method of undetermined multiplier - Problems only.
10. Infinite series of constant terms: Convergence and Divergence (definition). Cauchy's principle as applied to infinite series (application only). Series of positive terms: Statements of comparison test. D'AIembert's Ratio test. Cauchy's n-th root test and Raabe's test – applications. Alternating series. Statement of Leibnitz test and its applications.

Group – C [20 Marks]
(Linear Programming)

1. Mathematical Preliminaries: Euclidean Space, Linear dependence and Independence of vectors, Spanning set and Basis, Replacement of a vector in a Basis, Basic solution of a system of linear algebraic equations.
2. Slack and Surplus variables. Standard form of a L.P.P. and its matrix form. Feasible and Basic Feasible Solutions (B.F.S.). Degenerate and Non-degenerate B.F.S. Hyperplane, convex combination, line and line segment in E^n. Convex set, extreme points, convex hull and polyhedron.
3. Theorems (with proof): The set of all feasible solutions of a L.P.P. is a convex set. The objective function of a L.P.P. assumes its optimal value at an extreme point of the convex set of feasible solutions, a B.F.S. of a L.P.P. corresponds to an extreme point of the convex set of feasible solutions.
4. Fundamental Theorem of L.P.P. (Statement only). Reduction of a feasible solution to a B.F.S. Solution by simplex method and method of penalty.
5. Concept of Duality. The dual of the dual is the primal. Relation between the objective values of dual and the primal problems. Dual problems with at most one unrestricted variable, one constraint of equality.
6. Transportation and Assignment problems. Their optimal solutions.

References:

1. Sobhakar Ganguly and Manabendra Nath Mukherjee – A Treatise on Basic Algebra, Academic Publishers, Kolkata (3rd Edition)
2. S K Mapa – Higher Algebra (Abstract and Linear), Sarat Book House, Kolkata
3. B C Das and B N Mukherjee – Differential Calculus, U N Dhur & Sons Pvt. Ltd., Kolkata
4. K C Maity & R K Ghosh – Differential Calculus, New Central Book Agency (P) Ltd, Kolkata
5. J G Chakravorty & P R Ghosh –Linear Programming, Moulik Library Publisher, Kolkata
6. D C Sanyal & K Das – Linear Programming, U N Dhur & Sons Pvt. Ltd., Kolkata

<u>Question Pattern for End Semester (1 & 2) Examinations</u>

(Course Code: HMAT1GE01N & HMAT2GE01N & Paper: GE01)

Group – A (Modern Algebra, 15 marks)

(i) Answer (with reason) **two** objective / MCQ type questions of 2 marks (1 mark for correct option and 1 mark for reason) each from 4 given questions.

(ii) Answer **one** question of 3 marks from two given questions.

(iii) Answer **two** questions of 4 marks each from four given questions.

Group – B (Differential Calculus, 30 marks)

(i) Answer (with reason) **four** objective / MCQ type questions of 2 marks (1 mark for correct option and 1 mark for reason) each from 6 given questions.

(ii) Answer **two** questions of 3 marks each from four given questions.

(iii) Answer **four** questions of 4 marks each from six given questions.

Group – C (Linear Programming, 20 marks)

(i) Answer (with reason) **one** objective / MCQ type questions of 2 marks (1 mark for correct option and 1 mark for reason) each from two given questions.

(ii) Answer **three** questions of 6 marks each from five given questions. Each question may contain further parts.

SEMESTER – 3	
Name of the Paper: **GE02**	
Name of Course: Linear Algebra, Integral Calculus, Differential Equations, Laplace Transforms	
Course Code: HMAT3GE02N	
Full Marks: 100	Credit: 6
Theory	Credits : 5 Full Marks : 65+20*
Tutorial	Credits : 1 Full Marks : 15
Number of classes required : 100	
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance	

&

SEMESTER – 4	
Name of the Paper: **GE02**	
Name of Course: Linear Algebra, Integral Calculus, Differential Equations, Laplace Transforms	
Course Code: HMAT4GE02N	
Full Marks: 100	Credit: 6
Theory	Credits : 5 Full Marks : 65+20*
Tutorial	Credits : 1 Full Marks : 15
Number of classes required : 100	
*15 Marks are reserved for Internal Assessment (to be taken from the mid-semester exam of 30 marks) & 5 marks for Attendance	

Course Outcomes: At the end of studying this course a student will acquire

- fundamental knowledge and problem solving skills on certain topics on Linear Algebra mostly through matrix theory, which is of paramount importance as a useful mathematical tool for various subjects like Physics, Statistics, Economics etc.
- very important knowledge on integral calculus that will help the students to understand the applications of calculus to their respective major subjects.
- rudimentary knowledge and problem solving skill on certain classes of differential equations, Laplace transforms and their applications, which is indispensable as a tool for various other branches of science.

Detailed Syllabus

Group – A [30 Marks]
(Linear Algebra)

1. Concept of Vector space over a Field: Examples, Concepts of Linear combinations, Linear dependence and independence of a finite number of vectors, Sub- space, Concepts of generators and basis of a finite dimensional vector space. Problems on formation of basis of a vector space (No proof required).
2. Statement and applications of Deletion theorem, Extension theorem and Replacement theorem on finite dimensional vector spaces.
3. Real Inner product spaces, norms, statement of Cauchy-Schwarz's inequality, Gram-Schmidt orthonormalisation process, orthogonal basis (Stress should be given on solving problems).
4. Characteristic equation of square matrix of order not more than three; determinations of Eigen Values and Eigen Vectors (problems only). Statement and illustration of Cayley-Hamilton Theorem.
5. Linear transformations, matrix of a linear transformation, similarity and change of basis (General ideas and problems).
6. Diagonalization of matrix – problems only (relevant theorems to be stated).

Group – B [10 Marks]
(Integral Calculus)

1. Definition of Improper Integrals: Statements of (i) μ-test (ii) Comparison test (Limit from excluded) and simple problems only. Use of Beta and Gamma functions (convergence and important relations being assumed).
2. Working knowledge of multiple integrals.

Group – C [25 Marks]
(Differential Equations, Laplace Transforms)

1. First order differential equations: Exact equations and those reducible to such equations.
2. Linear differential equations and Bernoulli's equations. Equations reducible to Linear equations.
3. Clairaut's equations: General and Singular solutions.
4. Applications: Orthogonal Trajectories.
5. Second order linear differential equations with constant co-efficients (solution by operator method). Euler's Homogeneous equations.
6. Laplace Transform (LT): Statement of existence theorem of LT. Elementary properties of LT. Inverse Laplace Transform and its properties. Application to the solution of an ordinary differential equation (ODE) of second order with constant coefficients.

References:

1. Sobhakar Ganguly and Manabendra Nath Mukherjee – A Treatise on Basic Algebra, Academic Publishers, Kolkata (3rd Edition)
2. S K Mapa – Higher Algebra (Abstract and Linear), Sarat Book House, Kolkata
3. Stephen H Friedberg, Arnold J Insel and Lawrence E Spence – Linear Algebra, Prentice-Hall of India Pvt. Ltd., New Delhi (4th Edition, 2004)
4. B C Das and B N Mukherjee – Integral Calculus, U N Dhur & Sons Pvt. Ltd., Kolkata
5. K C Maity & R K Ghosh – Integral Calculus, New Central Book Agency (P) Ltd, Kolkata
6. K C Maity & R K Ghosh – Differential Equations, New Central Book Agency (P) Ltd, Kolkata
7. J G Chakravorty & P R Ghosh – Differential Equations, U N Dhur & Sons Pvt. Ltd., Kolkata
8. N Mandal & B Pal – Differential Equations (Ordinary and Partial) – Books and Allied Pvt Ltd, Kolkata
9. A Sarkar & N Mandal – A Course of Advanced Calculus, Joydurga Library, Kolkata.

Question Pattern for End Semester (3 & 4) Examinations

(Course Code: HMAT3GE02N & HMAT4GE02N & Paper: GE02)

Group – A (Linear Algebra, 30 marks)

(i) Answer (with reason) **three** objective / MCQ type questions of 2 marks (1 mark for correct option and 1 mark for reason) each from 2 given questions.

(ii) Answer **four** questions of 6 marks each from 6 given questions. Each question may contain further parts.

Group – B (Integral Calculus, 10 marks)

(i) Answer **two** objective / MCQ type questions of 1 mark each from three given questions.

(ii) Answer **two** questions of 4 marks each from 3 given questions. Each question may contain further parts.

Group – C (Differential Equations, Laplace Transforms, 25 marks)

(i) Answer **three** objective / MCQ type questions of 1 mark each from four given questions.

(ii) Answer **three** questions of 4 marks each from five given questions from Article 1 to 5. Each question may contain further parts.

(iii) Answer **two** questions of 5 marks each from three given questions from Article 6. Each question may contain further parts.