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Title: Conditionals and Contradiction in Paraconsistent Set Theory

Abstract: In a recent paper [4], Petersen presents new results based on a 2012 proof [6] of Cantor's theorem using paraconsistent set theory. Petersen suggests that these further (negative) results—contradictions that "threaten to multiply uncontrollably"—are a challenge to inconsistent (dialetheic) mathematics. In Petersen's proofs, key notions like identity and cardinality are defined using a material conditional in a Gentzen system version of the logic LP; he claims, contra the relevant logic used in [6], that a stronger conditional is not required. I will show that notions expressed using only the material conditional are much weaker than similar notions expressed in [6]; and this is why it is possible to prove 'too much' about e.g. cardinality when phrased only materially. I will connect this discussion to some other recent approaches to paraconsistent set theory [2, 3, 5; cf. 1], raise some open questions and challenges that arise from [4] for paraconsistent approaches like [7], and draw some conclusions about the importance of conditionals for paraconsistent mathematics.

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